

# Standard Model contributions to B and $B_s$ meson semileptonic decays

Heechang Na  
University of Utah

In collaboration with:

C. Bouchard, P. Lepage, C. Monahan, and J. Shigemitsu

**HPQCD collaboration**

Lattice 2014 at Columbia University, New York, USA

Jun-27-2014

# Outline

---

- HPQCD's heavy-light meson semileptonic decay programs
- **$B \rightarrow D \ell \nu$  and  $B_s \rightarrow D_s \ell \nu$  semileptonic decays**
  - Preliminary!
  - Dispersion relation
  - Chiral and continuum extrapolation
  - $R(D)$  and  $R(D_s)$
  - $f_0(B_s \rightarrow D_s)/f_0(B \rightarrow D)$ : important input for  $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$
  - alternative  $|V_{cb}|$  determination
- **$B_s \rightarrow K \ell \nu$  semileptonic decays**
  - First lattice calculation! C. Bouchard et al., arXiv:1406.2279
  - HPChPT z-expansion and error budget
  - Prediction of differential branching fractions
    - alternative  $|V_{ub}|$  determination
  - Phenomenology
    - $R_{\mu\tau}$ ,  $A_{FB}^l$ ,  $\bar{A}_{FB}^l$ , and  $A_{pol}^l$

# Heavy-light semileptonic projects from HPQCD

- $D \rightarrow K l\nu$ ,  $D \rightarrow \pi l\nu$ , and  $B \rightarrow K ll$  have been investigated recently.
  - PRD 82 (2010) 014506, PRD 84 (2011) 114505, PRD 88 (2013) 054509, PRL 111 (2013) 162002
- $B \rightarrow D l\nu$ ,  $B_s \rightarrow D_s l\nu$ , and  $B_s \rightarrow K l\nu$  for the Standard Model contribution in tree level

$$\langle X | V^\mu | B_x \rangle = f_+(q^2) (p_{B_x}^\mu + p_X^\mu - \frac{M_{B_x}^2 - M_X^2}{q^2} q^\mu) + f_0(q^2) \frac{M_{B_x}^2 - M_X^2}{q^2} q^\mu$$

$$q^2 = M_{B_x}^2 + M_X^2 - 2M_{B_x}E_X$$

$$\langle X | V^0 | B_x \rangle = \sqrt{2M_{B_x}} f_\parallel, \quad \langle X | V^k | B_x \rangle = \sqrt{2M_{B_x}} p_X^k f_\perp$$

$$f_0(q^2) = \frac{\sqrt{2M_{B_x}}}{M_{B_x}^2 - M_X^2} [(M_{B_x} - E_X) f_\parallel + (E_X^2 - M_X^2) f_\perp]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_x}}} [f_\parallel + (M_{B_x} - E_X) f_\perp]$$

# Heavy-light semileptonic projects from HPQCD

- We used MILC  $N_f=2+1$  asqtad gauge configurations

$\sim a$ (fm)	size	sea quarks	confs	tsrc
<b>0.12 (C1)</b>	24	0.005/0.05	2096 (1200)	2
<b>0.12 (C2)</b>	20	0.01/0.05	2256 (1200)	2
<b>0.12 (C3)</b>	20	0.02/0.05	1200 (600)	2
<b>0.09 (F1)</b>	28	0.0062/0.031	1896 (1200)	4
<b>0.09 (F2)</b>	28	0.0124/0.031	1200 (600)	4

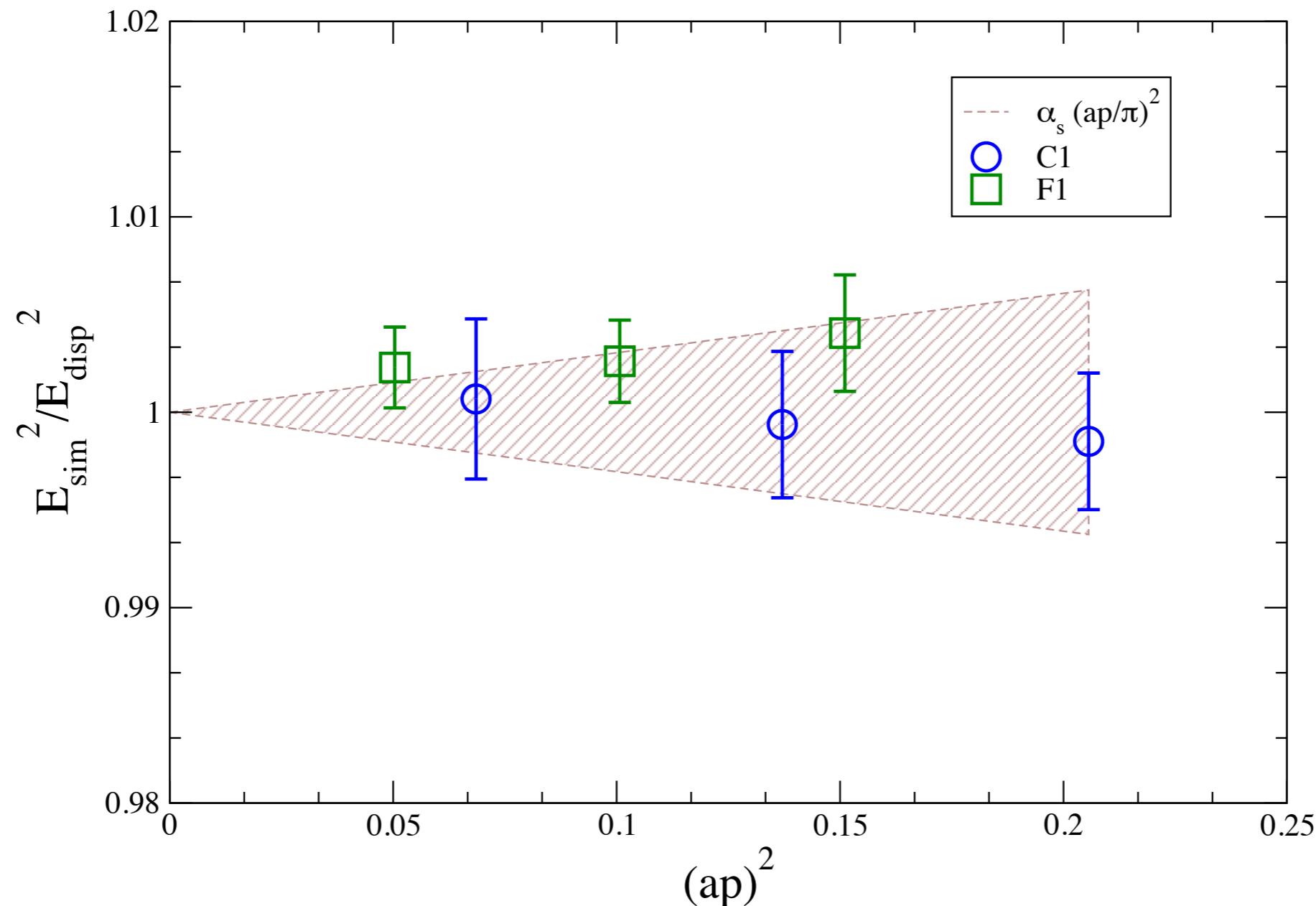
- We apply NRQCD heavy quark action for the bottom quark, and HISQ action for the light and charm quarks.
  - HISQ has leading discretization error at  $O(a_s(am_h)^2 v^2/c^2)$  and  $O((am_h)^4 v^2/c^2)$
  - The lattice vector current corrected through  $O(a_s, \Lambda_{\text{QCD}}/m_b, a_s/(am_b))$  in one loop perturbative calculation.
  - U(1) random-wall source
  - New chaining and marginalizing technique for correlator fits (arXiv:1406.2279)

# $B \rightarrow D$ and $B_s \rightarrow D_s$ semileptonic decays

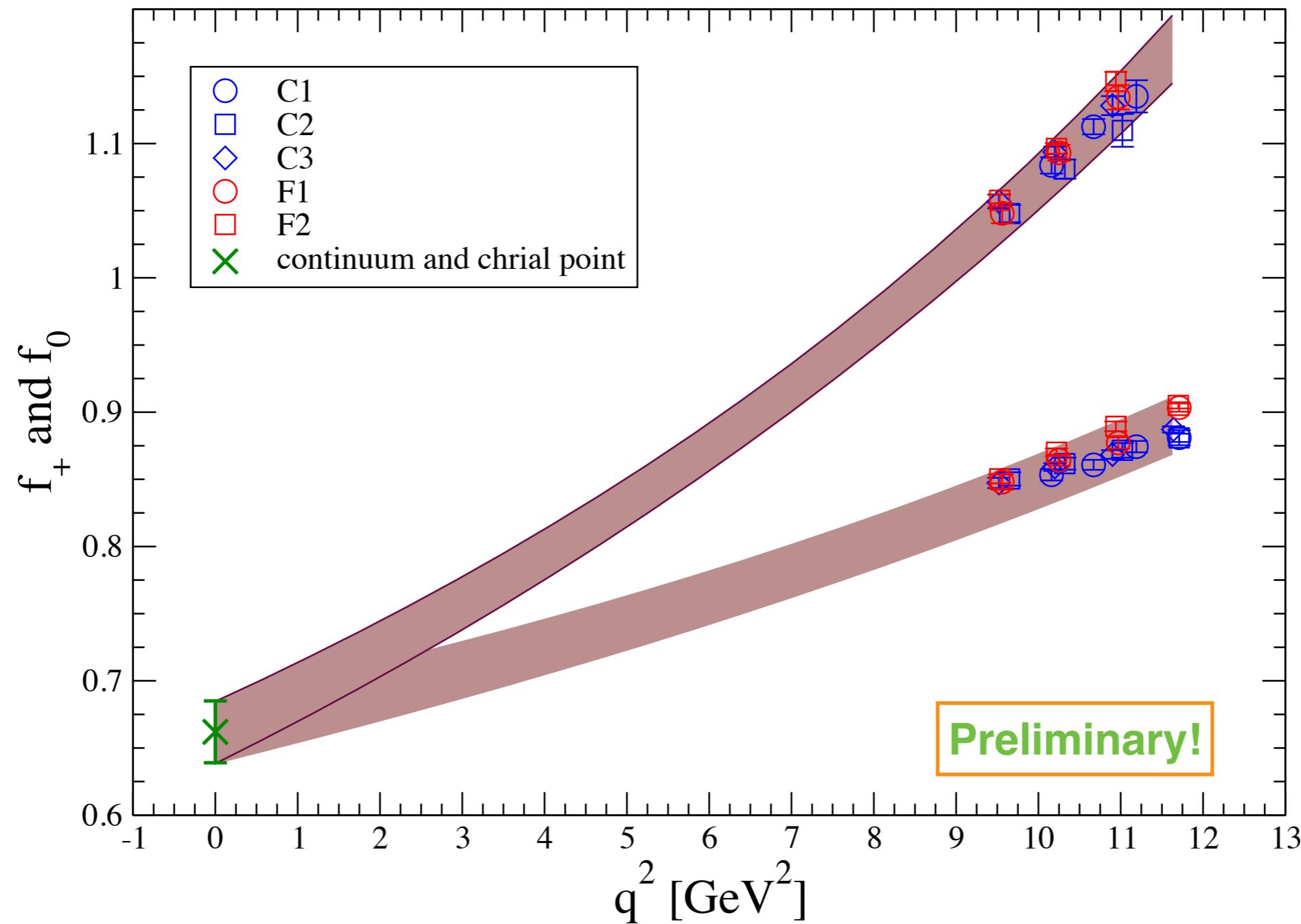
- Dispersion relation
- Chiral and continuum extrapolation
- $R(D)$  and  $R(D_s)$
- $f_0(B_s \rightarrow D_s)/f_0(B \rightarrow D)$ : important input for  $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$
- alternative  $|V_{cb}|$  determination
- **Preliminary!**

# ◆ Dispersion relation

- D mesons from HISQ light and charm quarks. Fully relativistic D meson.
- $E_{\text{sim}}^2/E_{\text{disp}}^2$  for D mesons on the coarse and fine ensembles



# After correlator fits...



# Modified z-expansion: Preliminary

- z-expansion with BCL parameterization

$$f_+(q^2) = \frac{1}{P} \sum_{k=0}^{K-1} a_k [z^k - (-1)^{k-K} \frac{k}{K} z^K], \quad P = 1 - \frac{q^2}{M_{B_c^*}}$$

- Modified z-expansion for continuum, chiral, and kinematic extrapolation

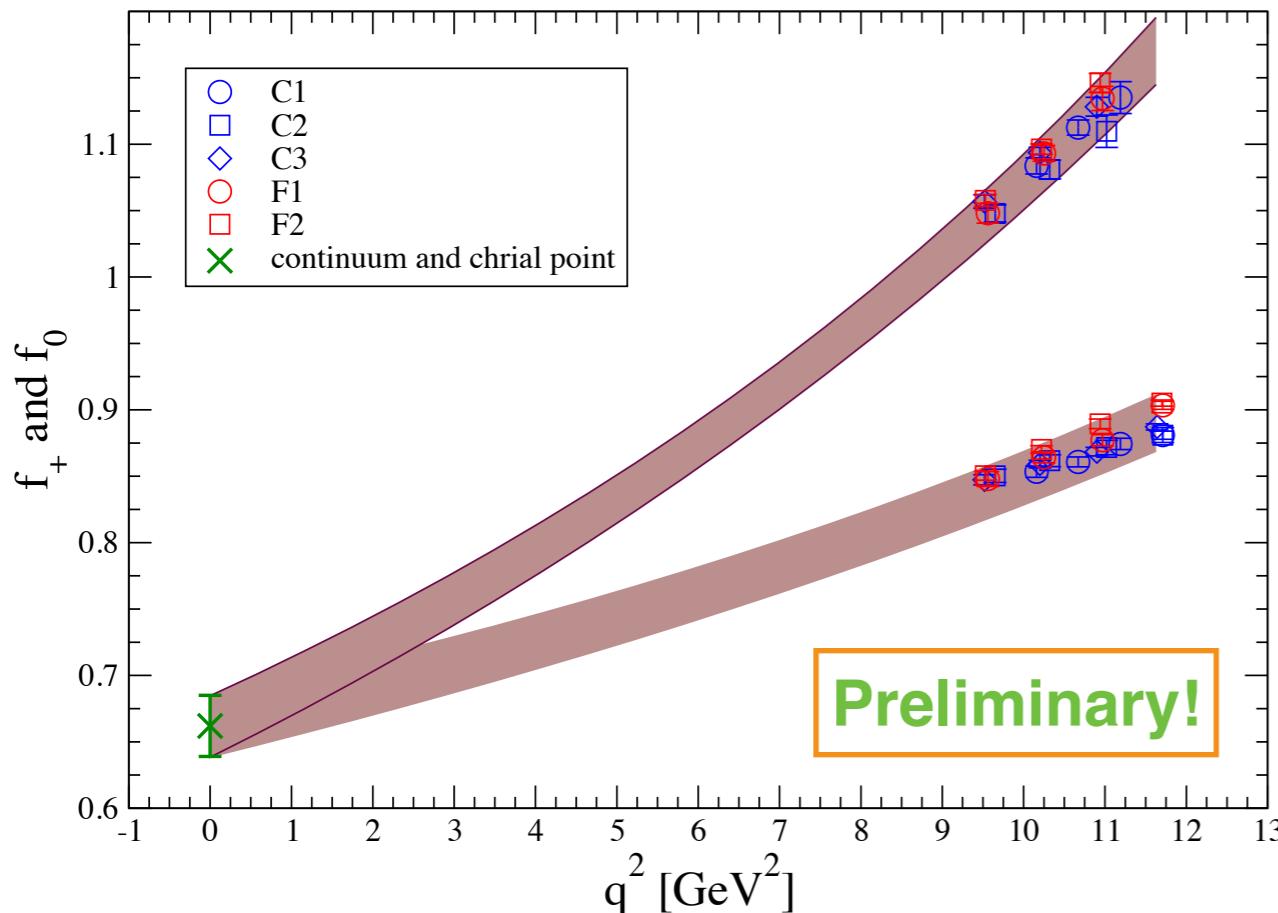
$$f_+(q^2) = \frac{1}{P} \sum_{k=0}^{K-1} a_k \mathbf{D}_k [z^k - (-1)^{k-K} \frac{k}{K} z^K]$$

$$D_k = 1 + c_1 x_\pi + c_2 \left( \frac{1}{2} \delta x_\pi + \delta x_K \right) + c_3 x_\pi \log x_\pi + d_1 (am_c)^2 + d_2 (am_c)^4 + e_1 (aE_D/\pi)^2 + e_2 (aE_D/\pi)^4$$

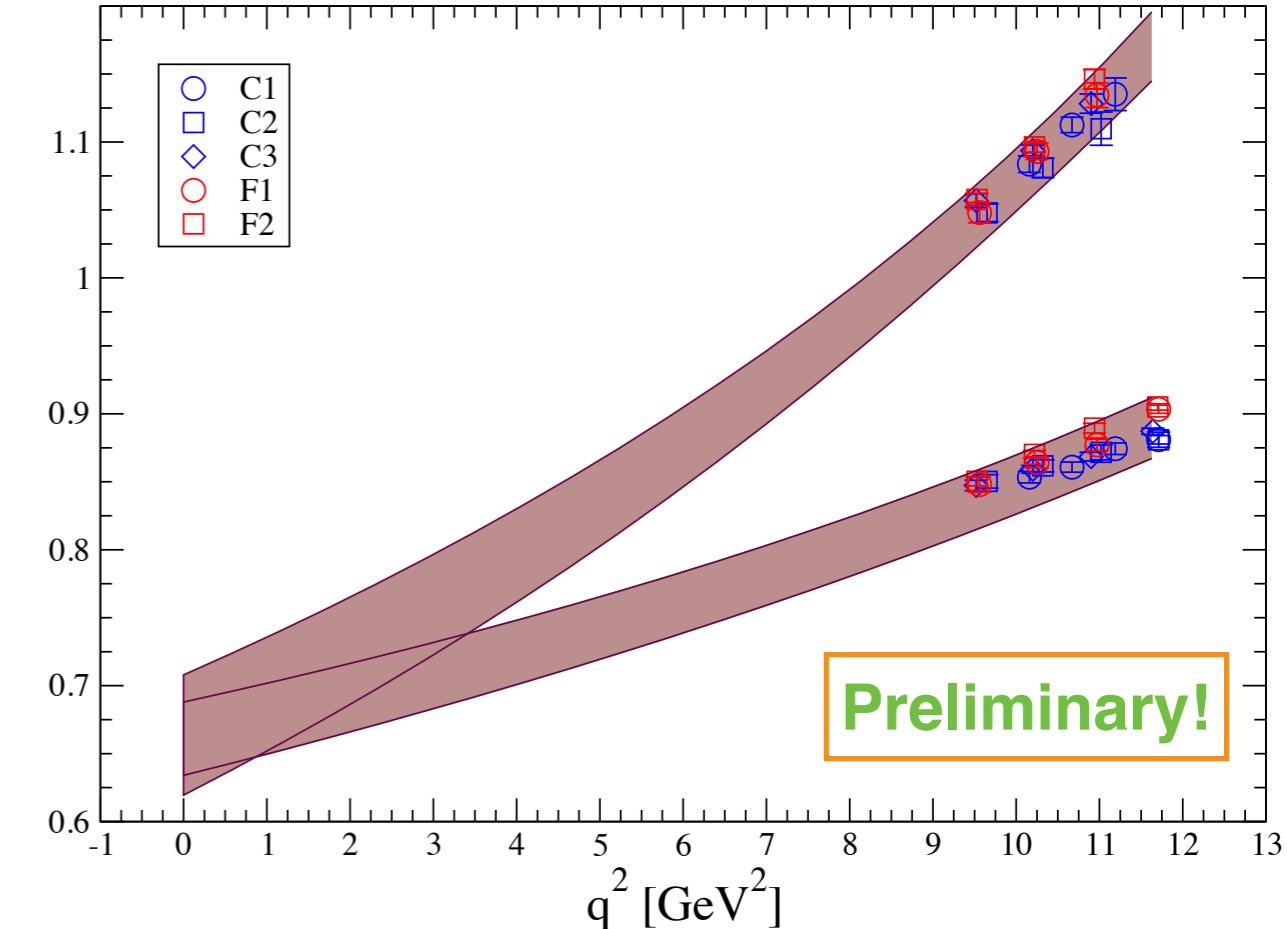
- Pole locations
  - $f_+$ :  $B_c^*$  ( $1^-$ ) = 6.33 GeV from a lattice calculation by HPQCD (PRL 104 (2010) 022001)
  - $f_0$ : No measurement or lattice calculations for  $B_c$  ( $0^+$ ).
    - We used  $6.33 + \exp(\log(0.2 \pm 1.0))$  GeV.

# ◆ Modified z-expansion

- $B \rightarrow D \bar{D} \nu$



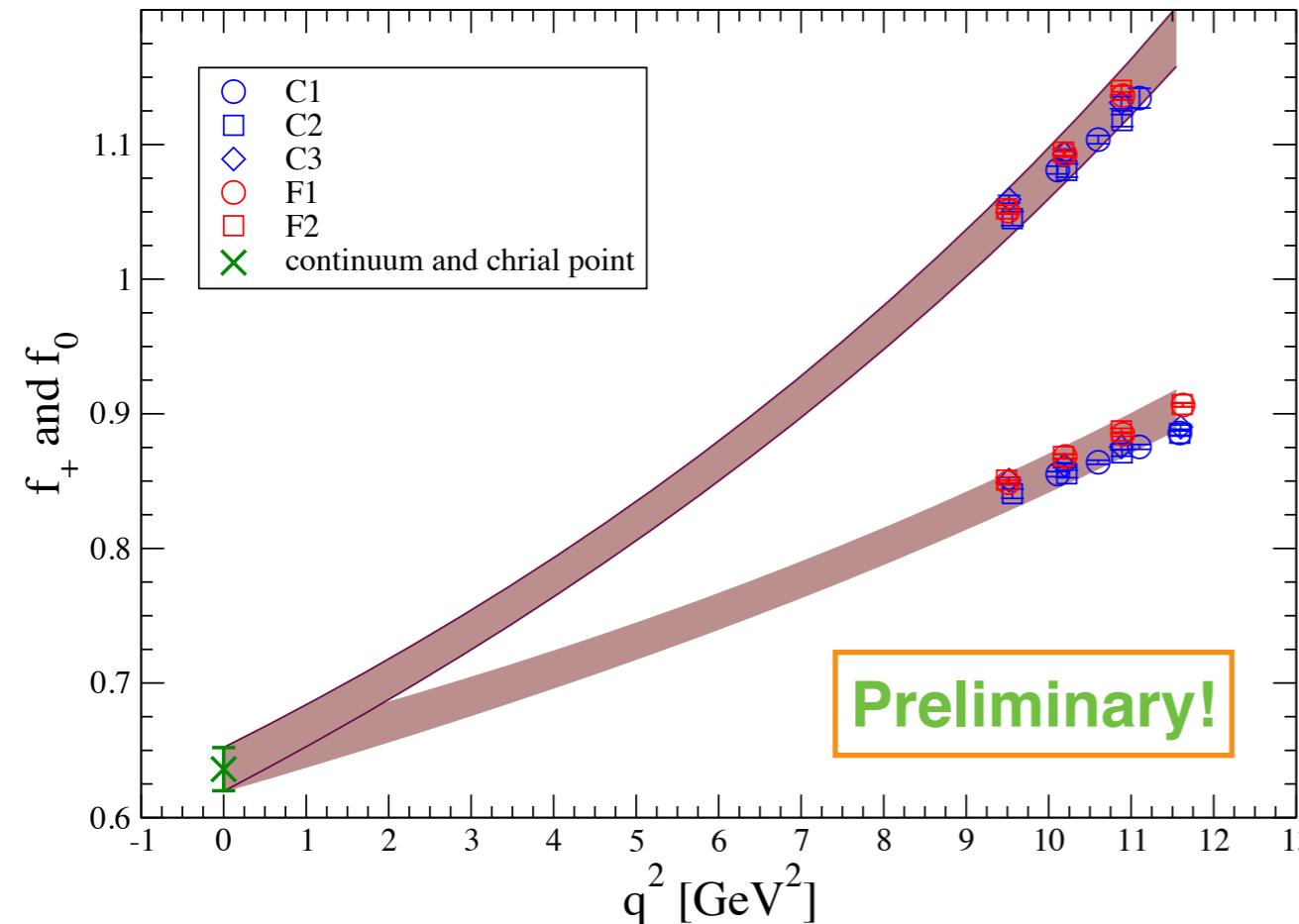
$\chi^2/\text{dof} = 0.76, Q = 0.85$   
With kinematic constraint:  $f_+(0) = f_0(0)$



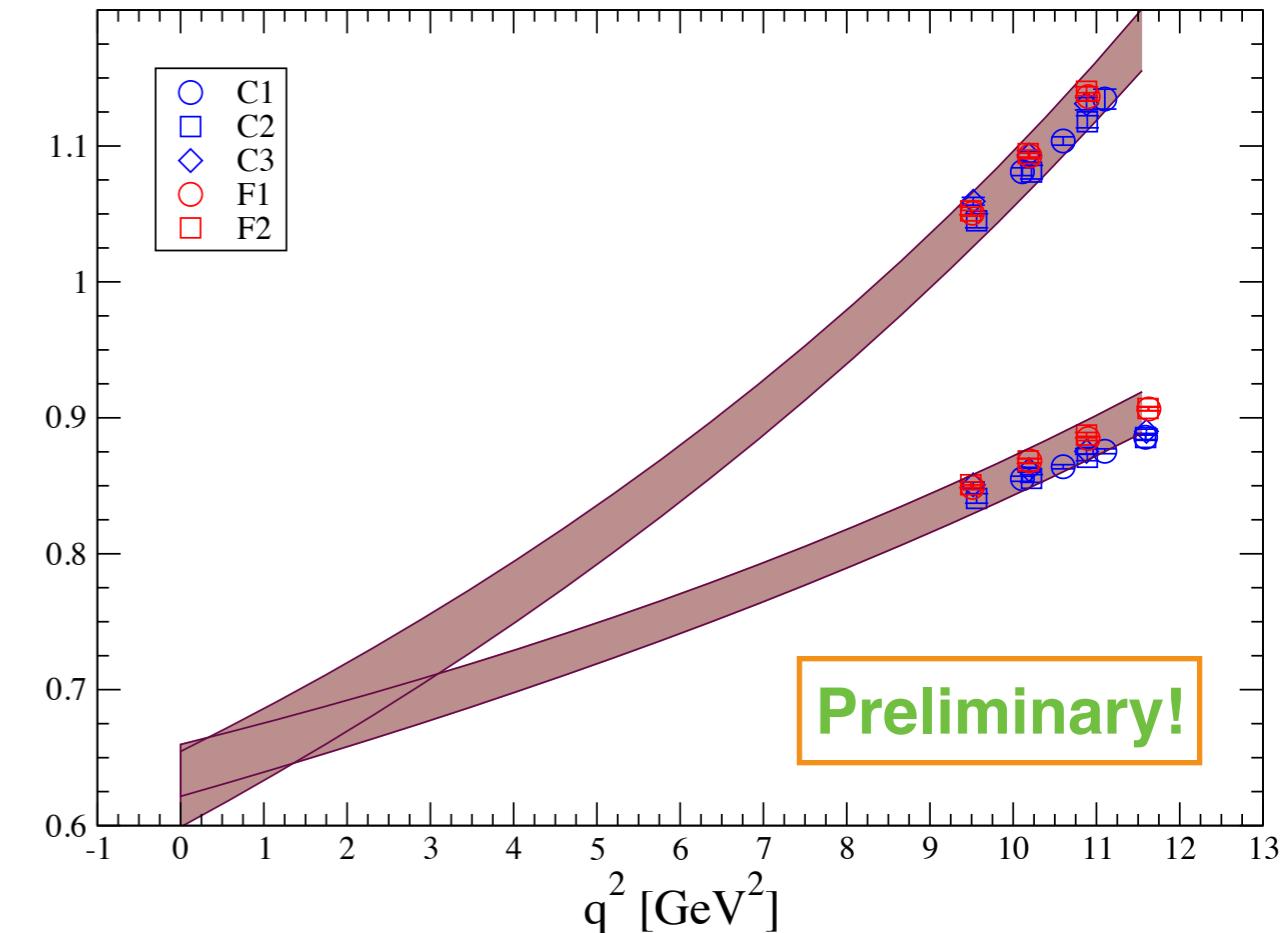
$\chi^2/\text{dof} = 0.78, Q = 0.82$   
Without kinematic constraint

# ◆ Modified z-expansion

- $B_s \rightarrow D_s \bar{l} \nu$



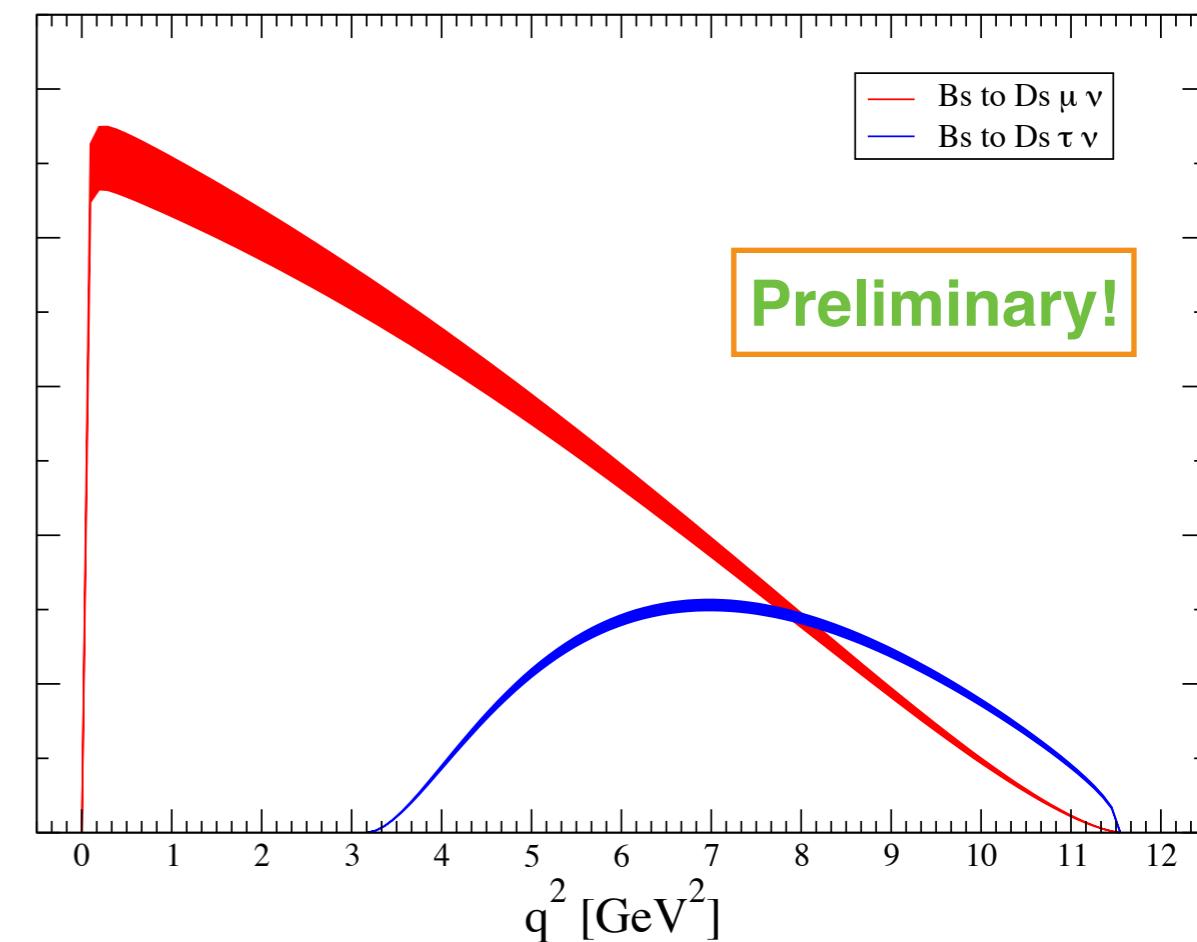
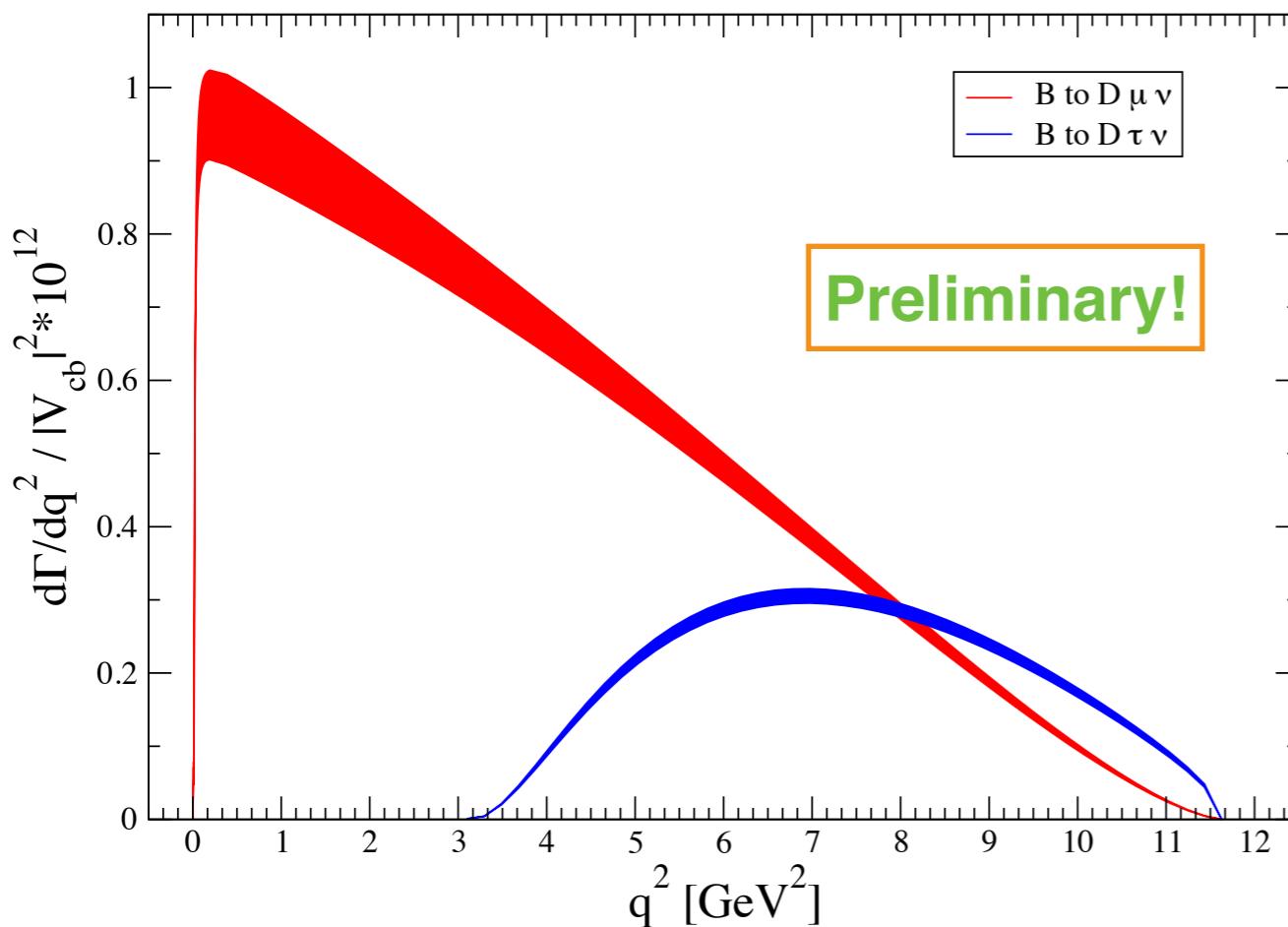
chi<sup>2</sup>/dof = 0.78, Q = 0.83  
With kinematic constraint



chi<sup>2</sup>/dof = 0.79, Q = 0.8  
Without kinematic constraint

- R(D) and R( $D_s$ ): 
$$R(D_x) = \frac{BR(B_x \rightarrow D_x \tau \nu)}{BR(B_x \rightarrow D_x \mu \nu)}$$

- Potential New Physics signature:  $B_x \rightarrow D_x \tau \nu$  and  $B_x \rightarrow D_x \mu \nu$ 
  - The scalar form factor is more important for  $B_x \rightarrow D_x \tau \nu$
- $B \rightarrow D$  and  $B_s \rightarrow D_s$  show almost identical results.
  - Kinematic variable,  $q^2$ , is very close.  $q^2 = m_B^2 + m_D^2 - 2m_B E_D$



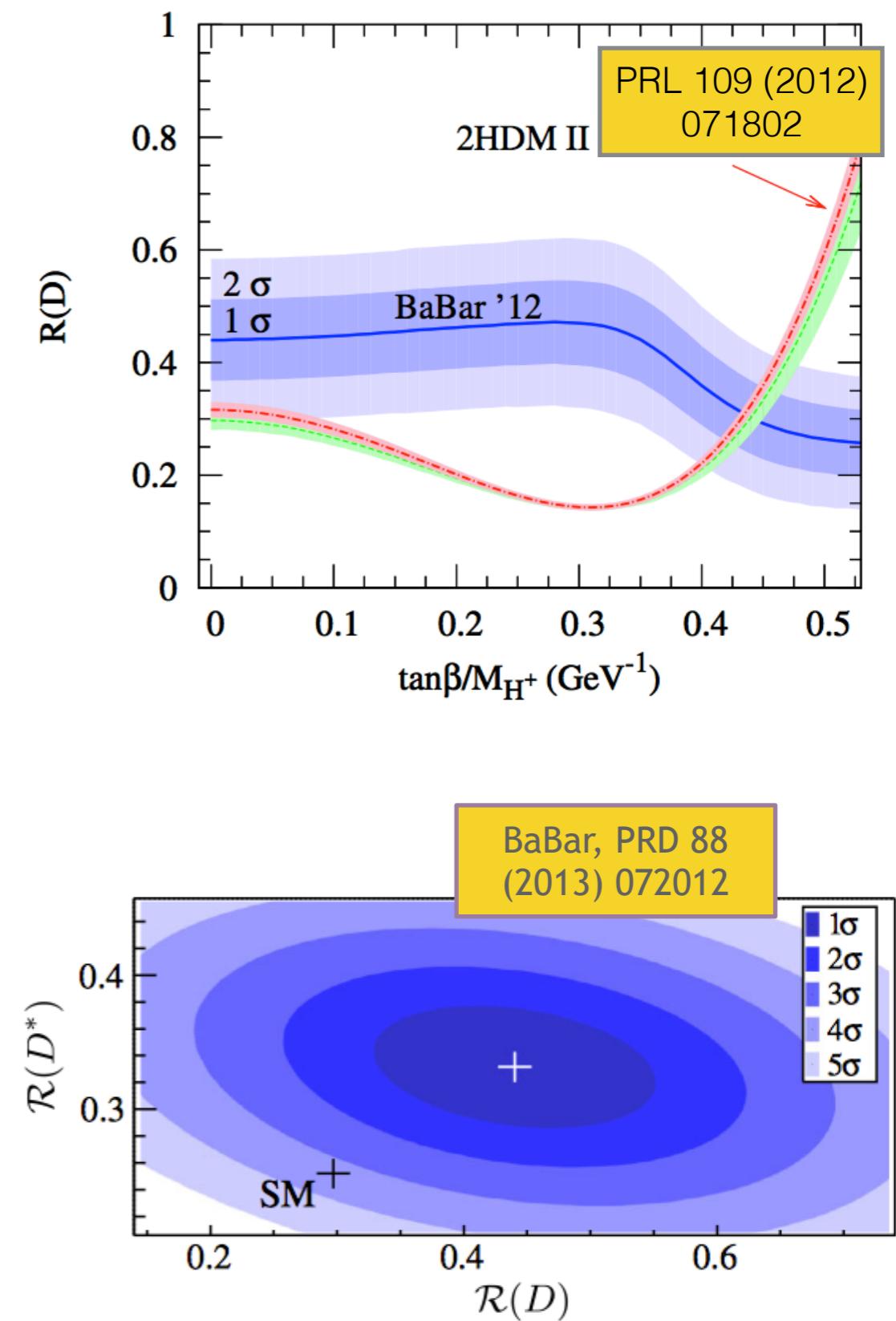
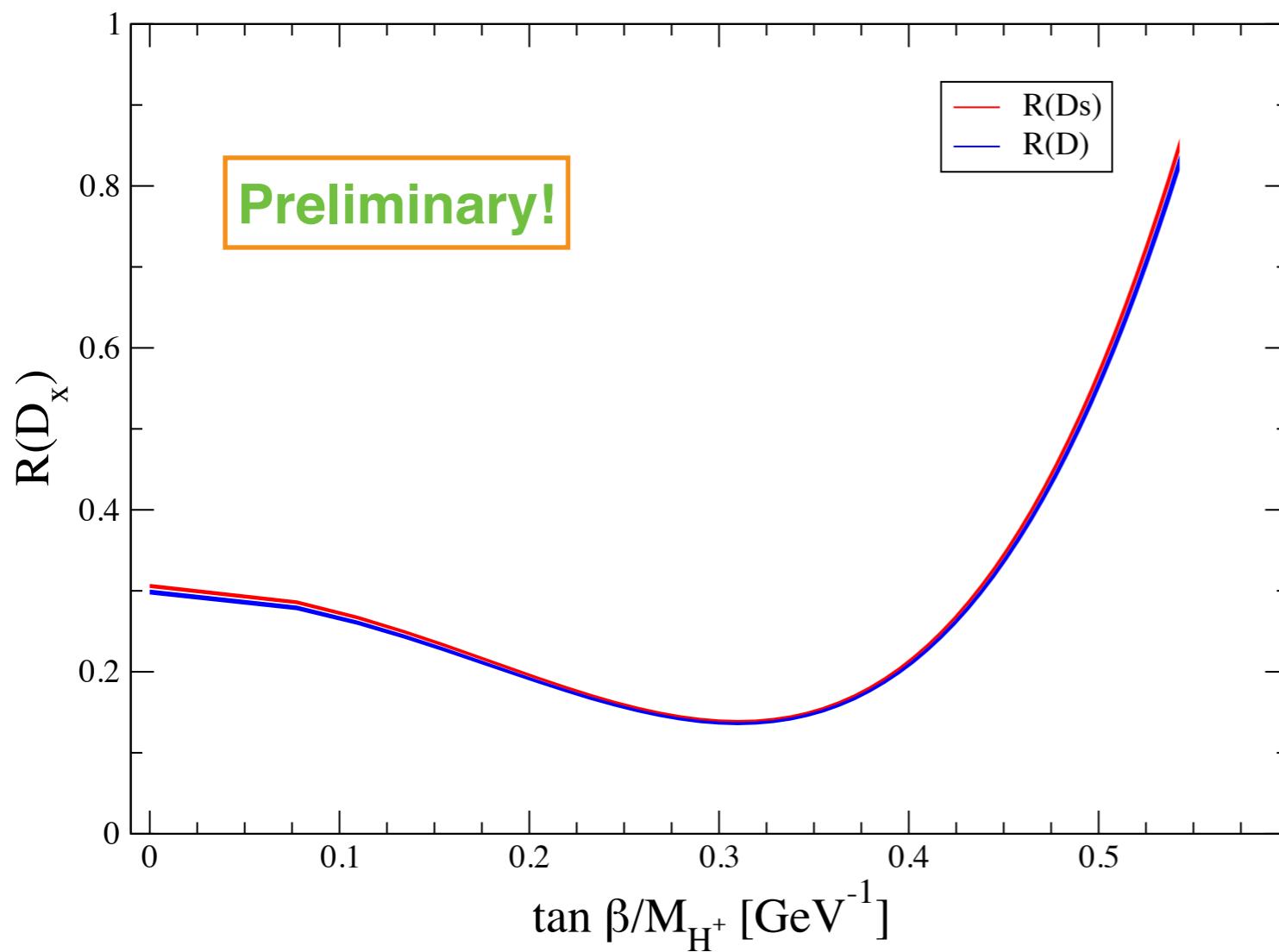
**B → D channels**

**$B_s \rightarrow D_s$  channels**

# R(D) and R(D<sub>s</sub>): 2HDM

- From 2HDM, the scalar coupling is given by

$$G_S \propto G_F V_{cb} m_l \frac{\tan^2 \beta}{M_{H^\pm}^2}$$



◆  $f_0(B_s \rightarrow D_s; m_\pi^2)/f_0(B \rightarrow D; m_K^2)$ : important input for  $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$

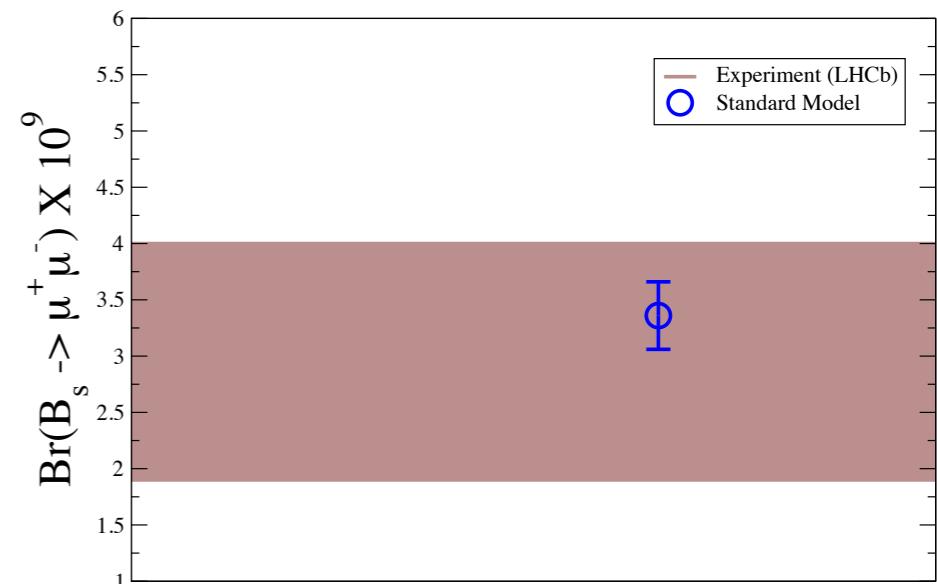
- Probing New Physics from  $B_s \rightarrow \mu^+ \mu^-$

- Highly suppressed in SM

$$Br^{SM}(B_s \rightarrow \mu^+ \mu^-) = (3.36 \pm 0.30) \times 10^{-9}$$

$$Br^{EXP}(B_s \rightarrow \mu^+ \mu^-) = 2.9_{-1.0}^{+1.1} \times 10^{-9}$$

- LHCb, PRL 111 (2013) 101805



- How to measure the branching ratio in experiments?

$$Br(B_s \rightarrow \mu^+ \mu^-) = Br(B_s \rightarrow X) \frac{\epsilon_X}{\epsilon_{\mu\mu}} \frac{N_{\mu\mu}}{N_X}$$

→  $Br(B_s \rightarrow \mu^+ \mu^-) = Br(B_d \rightarrow X) \frac{f_d}{f_s} \frac{\epsilon_X}{\epsilon_{\mu\mu}} \frac{N_{\mu\mu}}{N_X}$

- Now,  $\frac{f_d}{f_s}$  is the dominant source of the systematic errors!

◆  $f_0(B_s \rightarrow D_s; m_\pi^2)/f_0(B \rightarrow D; m_K^2)$ : important input for  $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$

- Combined result (LHCb-CONF-2013-011):

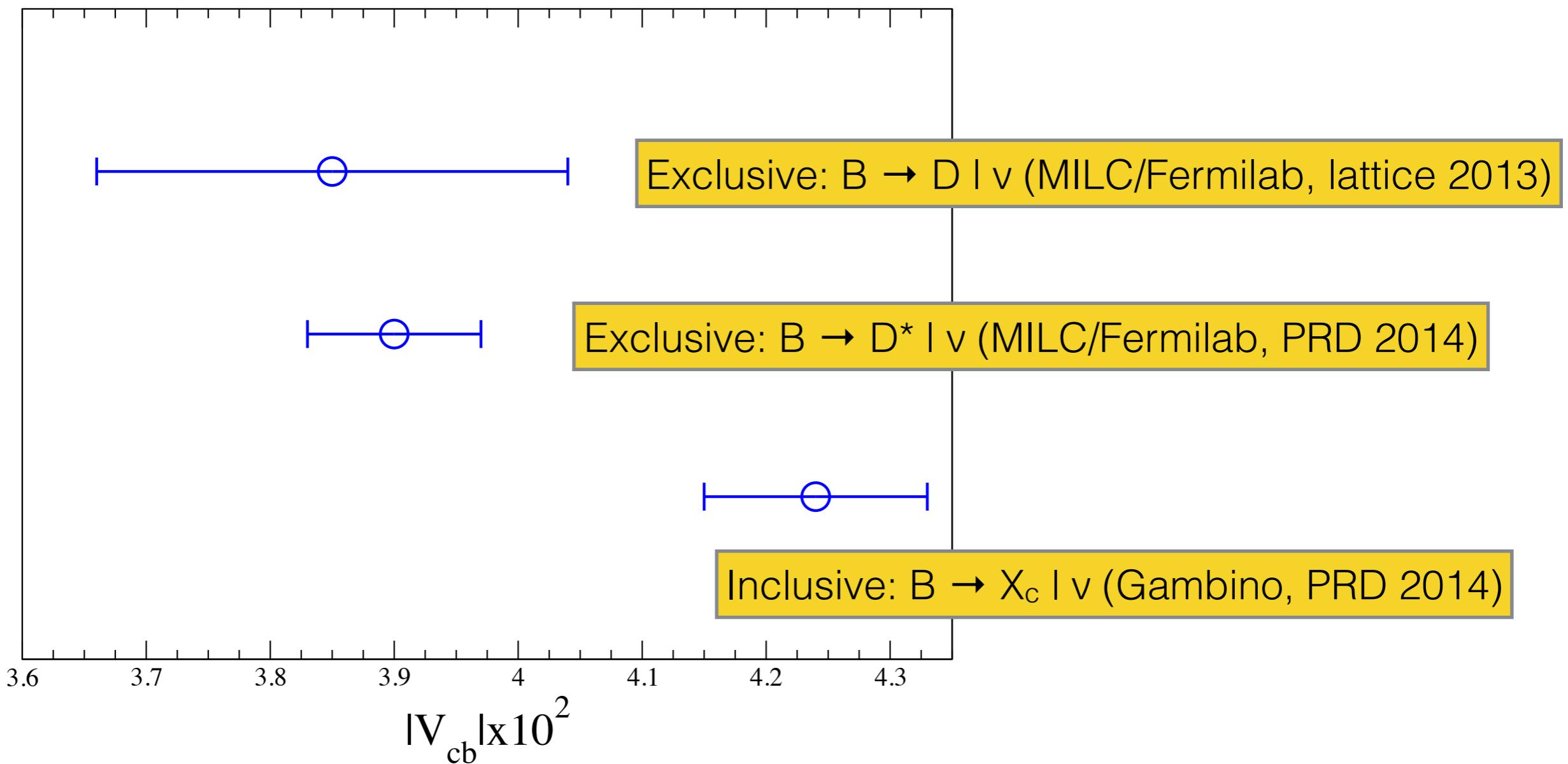
$$\frac{f_s}{f_d} = 0.259 \pm 0.015$$

$$\xrightarrow{\quad} + \begin{aligned} \left( \frac{f_s}{f_d} \right)_{semi} &= 0.263 \pm 0.008(stat)^{+0.019}_{-0.016}(syst) \\ \left( \frac{f_s}{f_d} \right)_{hadr} &= 0.242 \pm 0.004(stat) \pm 0.012(syst) \pm 0.021(theo) \end{aligned}$$

- The theory errors are came from the ratio of the form factors,  $N_F$ .
- $N_F = 1.092(93)$  from Fermilab/MILC, PRD 86 (2012) 039904
  - $N_F = 1.24 \pm 0.08$  (Sum rule, Blasi et al PRD 49 (1994) 238)
- We are getting the result soon with at least 30% smaller errors!
  - Need to take into account the correlations
- More independent lattice calculations are needed!
- $\frac{f_d}{f_s}$  will be used for all other branching ratio experiments with  $B_s$ 
  - Even for the CMS experiments

# $|V_{cb}|$

- $B \rightarrow D$  and  $B_s \rightarrow D_s$  can be used for alternative determinations of  $|V_{cb}|$ 
  - But, the most accurate determination is from  $B \rightarrow D^* l \bar{\nu}$  at zero recoil, mostly due to the experiments.



# $B_s \rightarrow K$ semileptonic decays

Lead by Chris Bouchard  
arXiv: 1406.2279

- First lattice calculation!
- HPChPT z-expansion and error budget
- Prediction of differential branching fractions
  - alternative  $|V_{ub}|$  determination
- Phenomenology
  - $R_{\mu\tau}$ ,  $A_{FB}^l$ ,  $\bar{A}_{FB}^l$ , and  $A_{pol}^l$

# HPChPT z-expansion

- HPChPT-motivated modified z-expansion

$$f_{\parallel, \perp}(E) = (1 + [\text{logs}]) \mathcal{K}_{\parallel, \perp}(E)$$

$$f_+(q^2) = \frac{1}{P} \sum_{k=0}^{K-1} a_k [z^k - (-1)^{k-K} \frac{k}{K} z^K] \quad \longrightarrow \quad f_+(q^2) = \frac{1}{P} \sum_{k=0}^{K-1} a_k \mathbf{D}_k [z^k - (-1)^{k-K} \frac{k}{K} z^K]$$

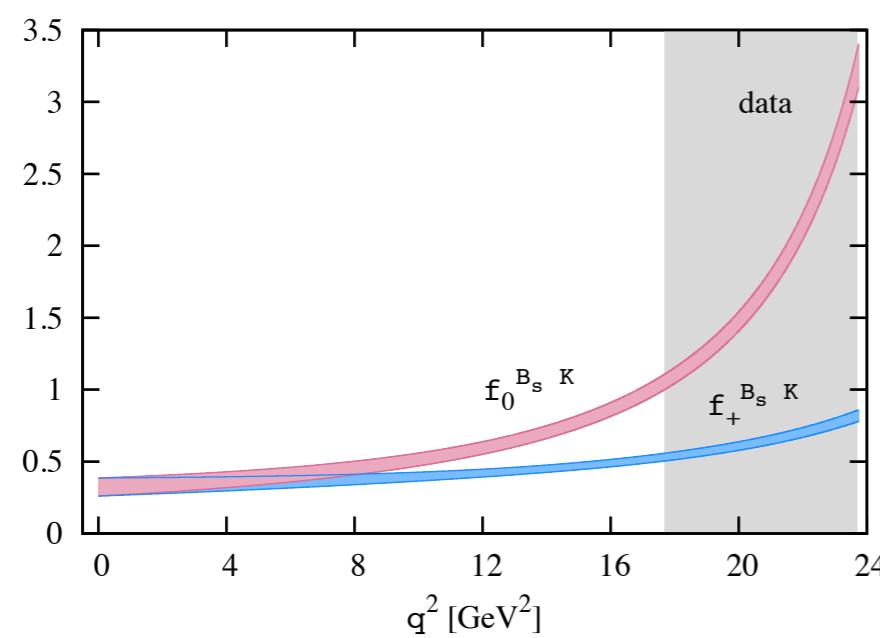
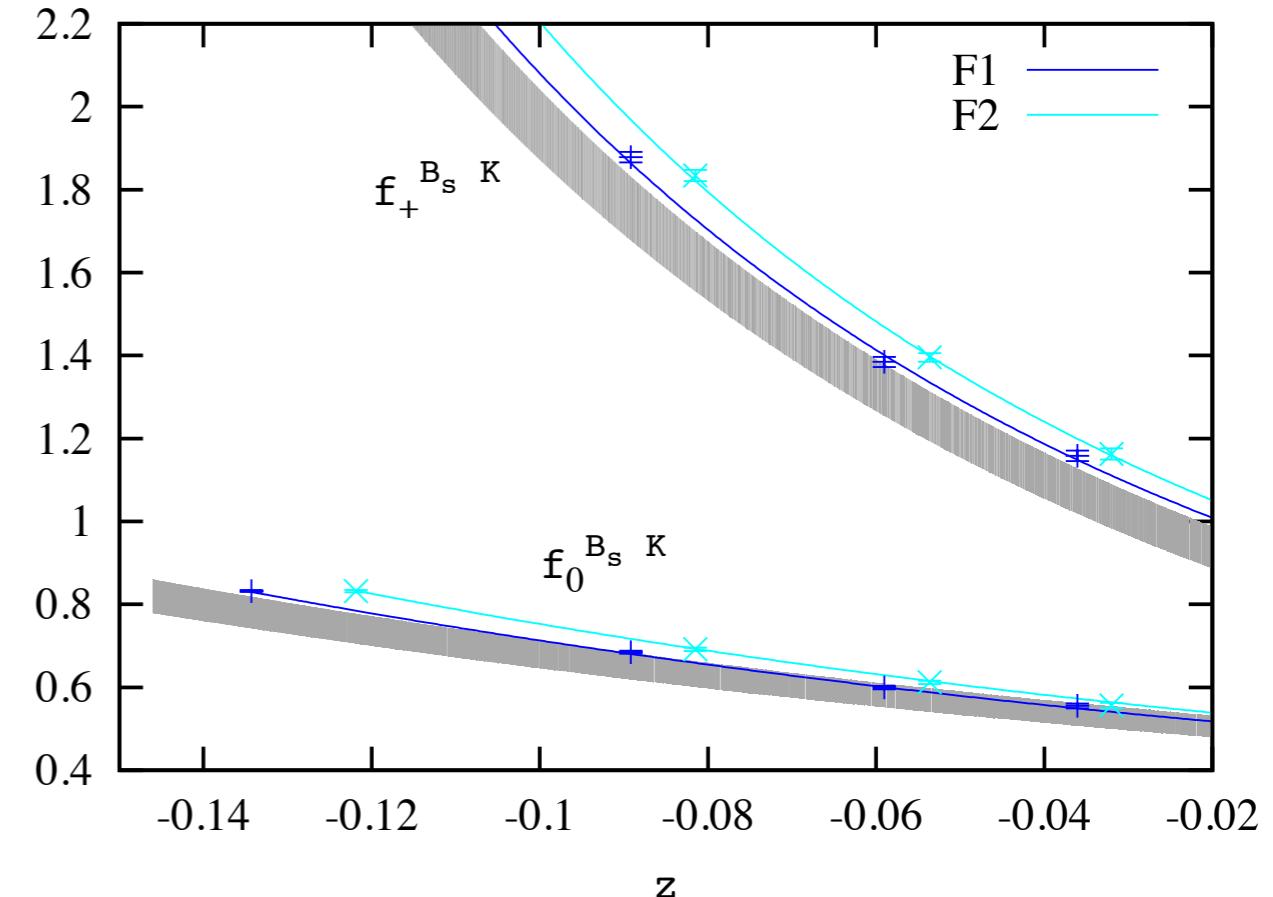
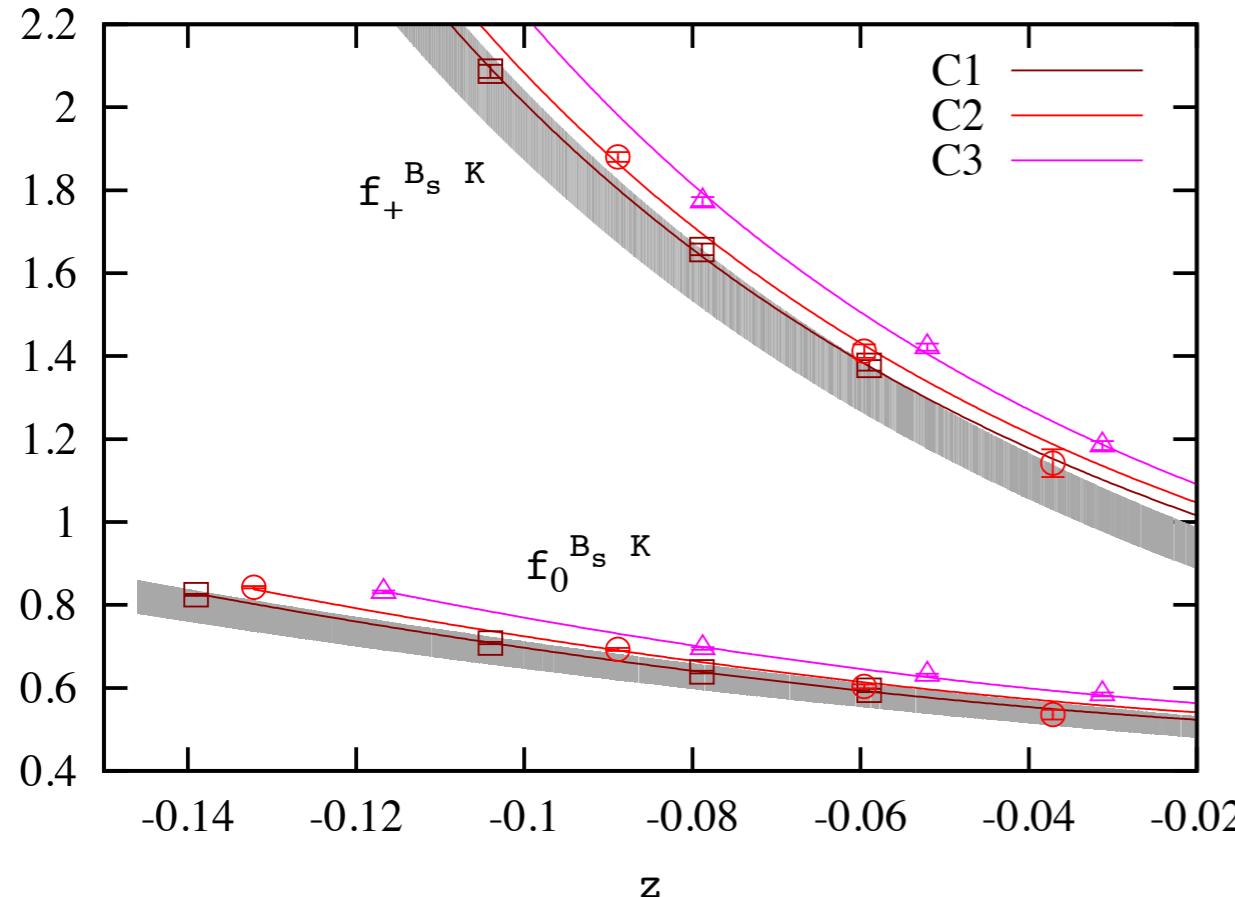
$$\longrightarrow \quad f_+(q^2) = (1 + [\text{logs}]) \frac{1}{P} \sum_{k=0}^{K-1} a_k D_k [z^k - (-1)^{k-K} \frac{k}{K} z^K]$$

$$\begin{aligned} D_k &= 1 + c_1^{(k)} x_\pi + c_2^{(k)} \left( \frac{1}{2} \delta x_\pi + \delta x_K \right) \\ &+ c_3^{(k)} \delta x_{\eta_s} + d_1^{(k)} (a/r_1)^2 + d_2^{(k)} (a/r_1)^4 \\ &+ e_1^{(k)} (a E_K)^2 + e_2^{(k)} (a E_K)^4, \\ [\text{logs}] &= -\frac{3}{8} x_\pi (\log x_\pi + \delta_{FV}) - \frac{1+6g^2}{4} x_K \log x_K \\ &- \frac{1+12g^2}{24} x_\eta \log x_\eta, \end{aligned}$$

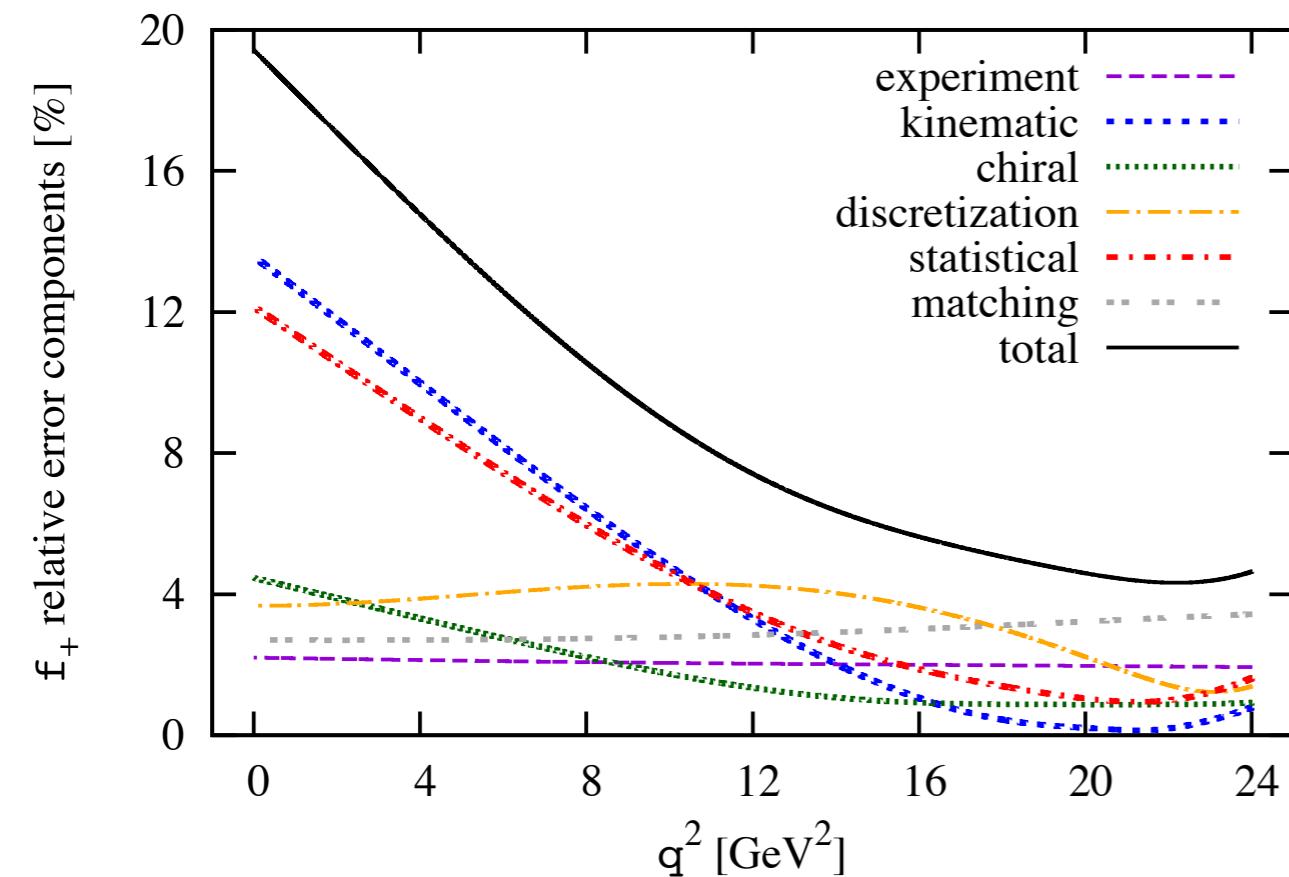
$$\begin{aligned} d_1^{(k)} &\rightarrow d_1^{(k)} (1 + l_1^{(k)} x_\pi + l_2^{(k)} x_\pi^2) (1 + h_1^{(k)} \delta x_b + h_2^{(k)} \delta x_b^2) \\ d_2^{(k)} &\rightarrow d_2^{(k)} (1 + l_3^{(k)} x_\pi + l_4^{(k)} x_\pi^2) (1 + h_3^{(k)} \delta x_b + h_4^{(k)} \delta x_b^2) \end{aligned}$$

# HPChPT z-expansion

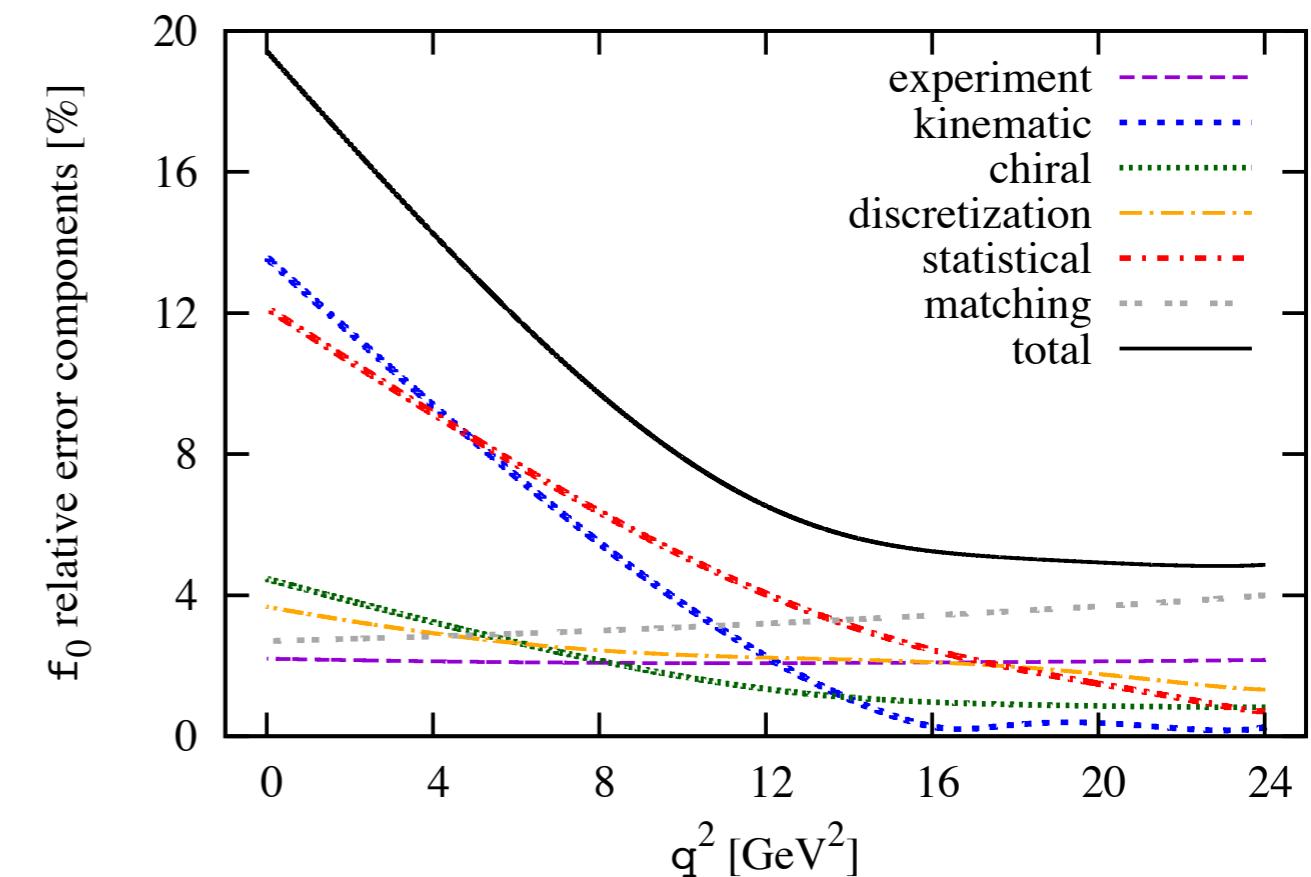
- Chiral and continuum extrapolation results



# Form factors: error budget



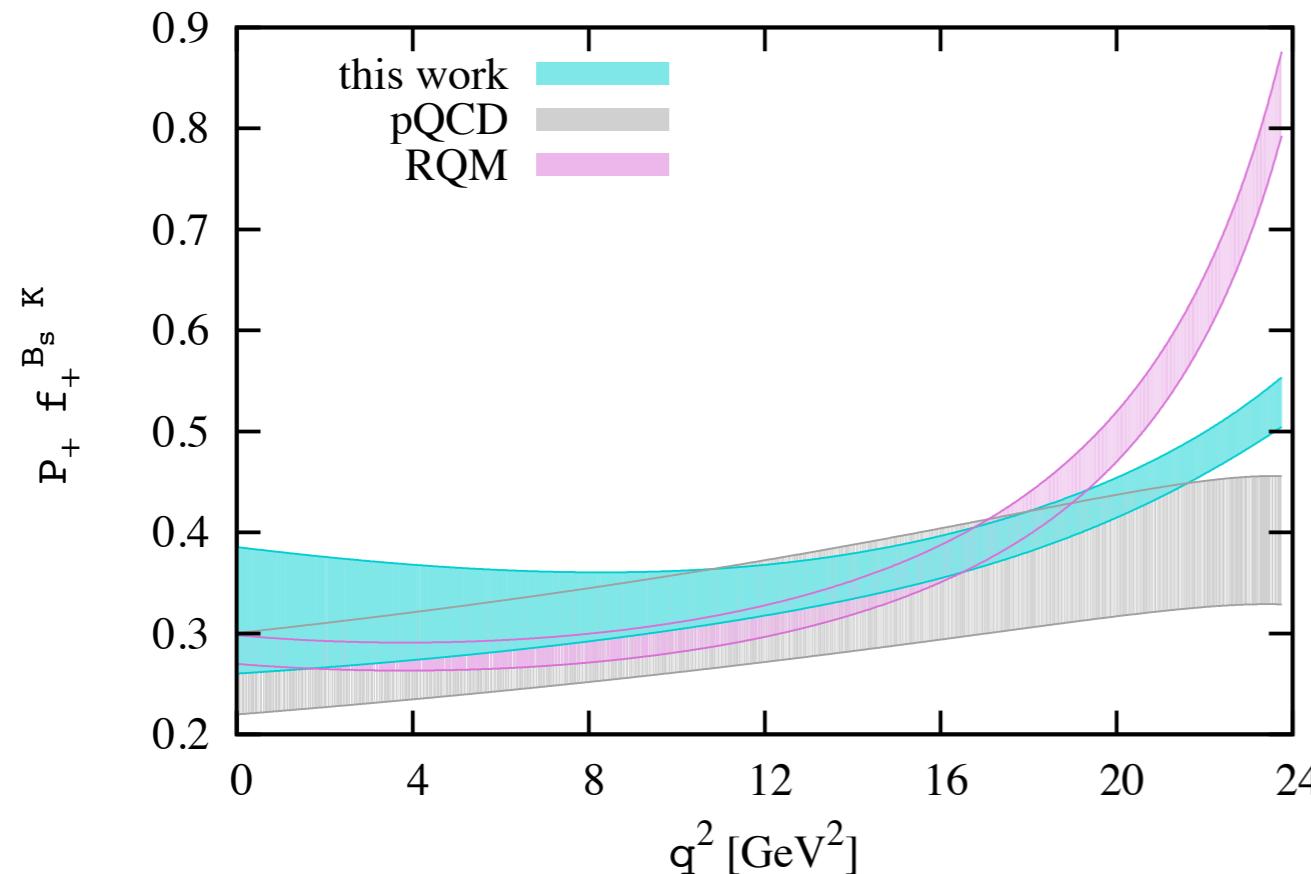
$\text{Bs} \rightarrow \text{K}, f_+$



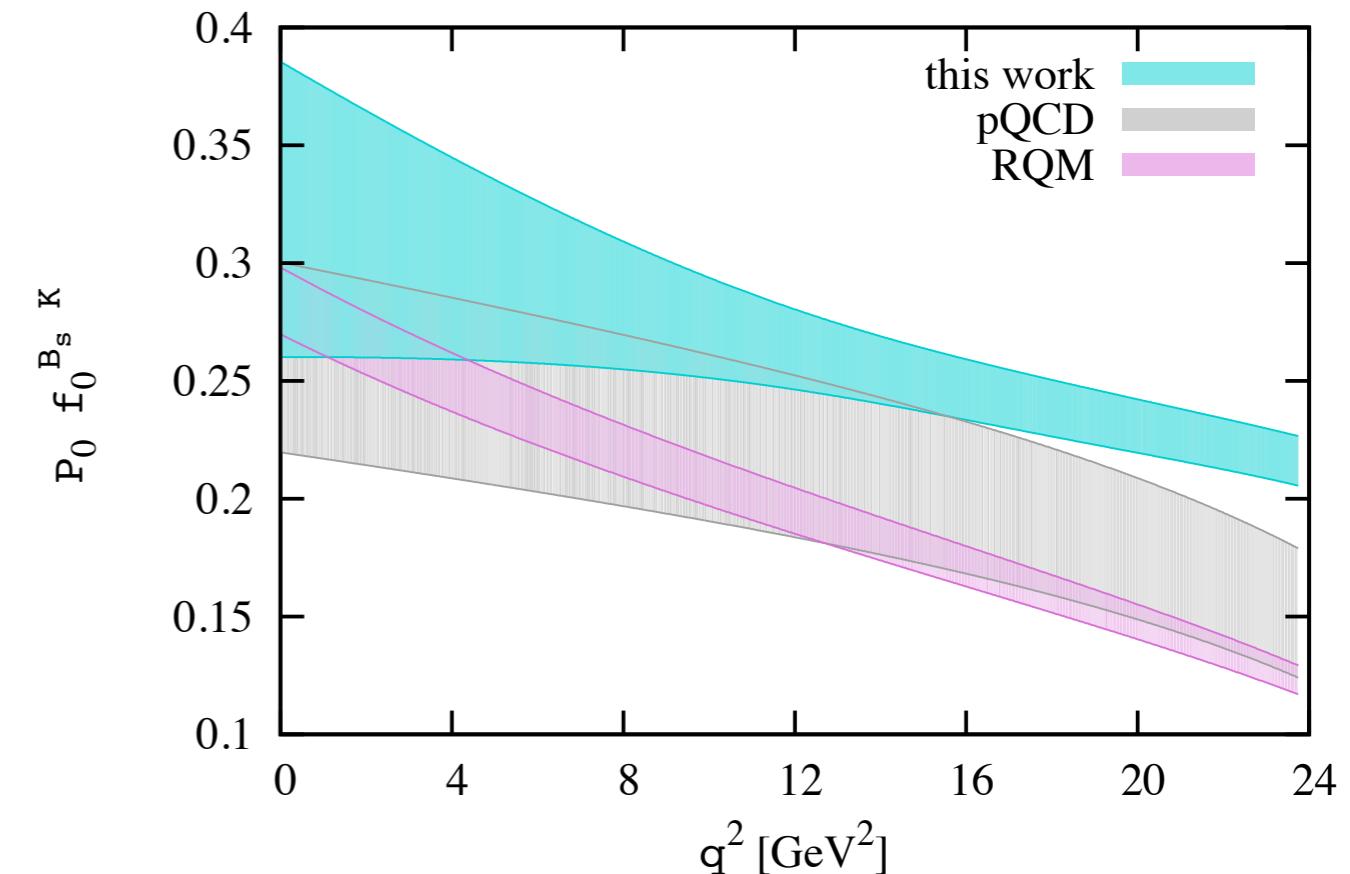
$\text{Bs} \rightarrow \text{K}, f_0$

- $f_+(q^2=0) = 0.323(63)$
- Kinematic and statistical errors are the dominant errors at small  $q^2$

# Form factors: comparisons to non-lattice calculations



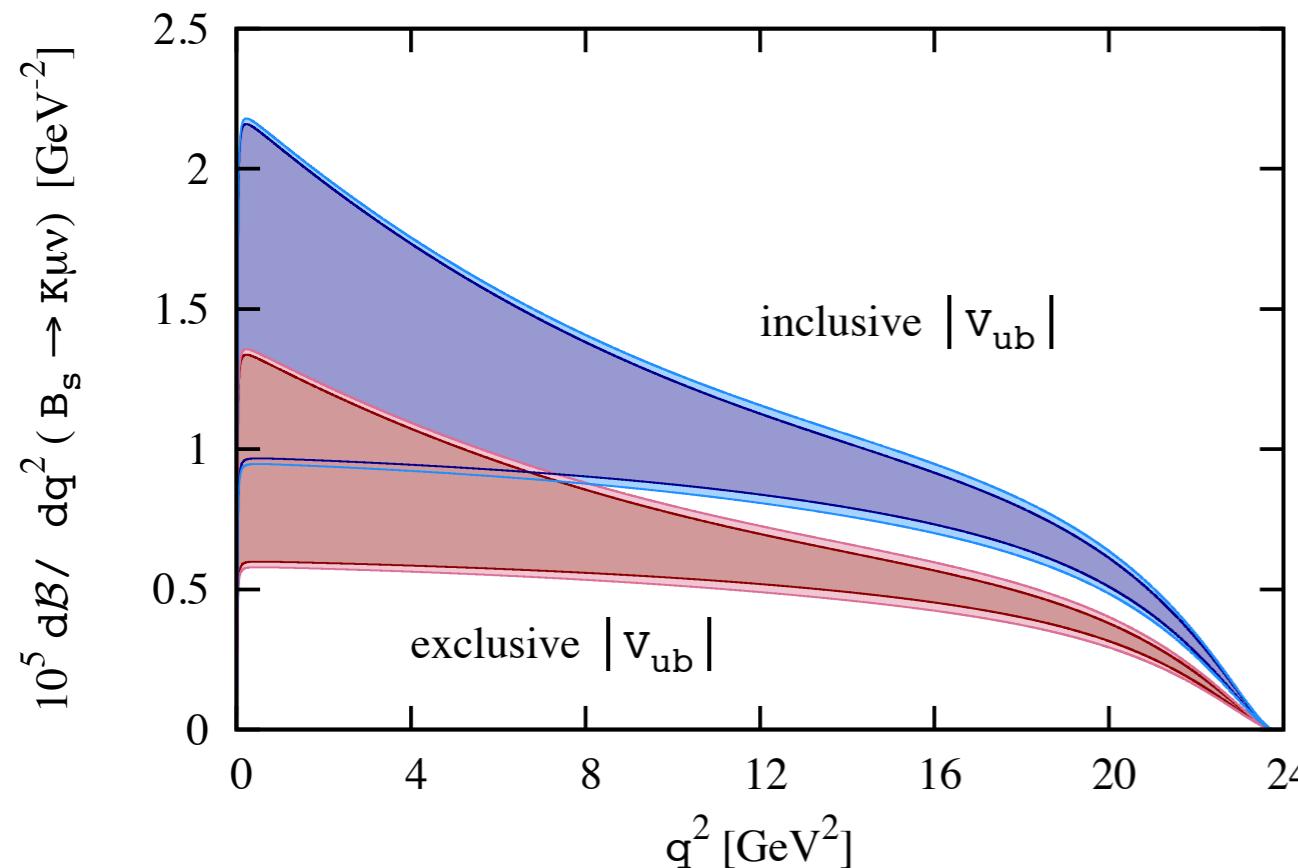
$B_s \rightarrow K, f_+$



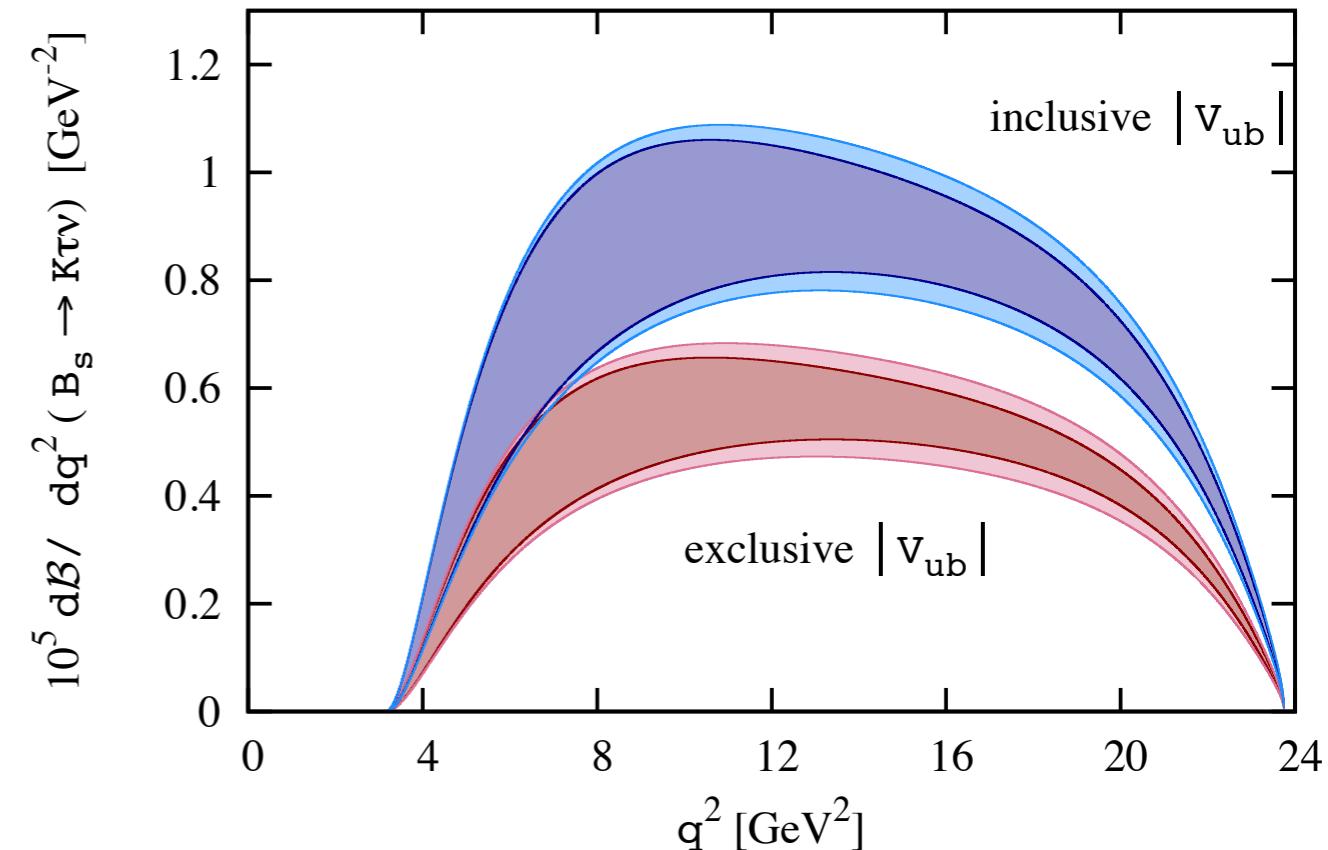
$B_s \rightarrow K, f_0$

- pQCD: perturbative QCD
- ROM : Relativistic Quark Model
- much better clarification at large  $q^2$

# Branching fractions: Predictions!

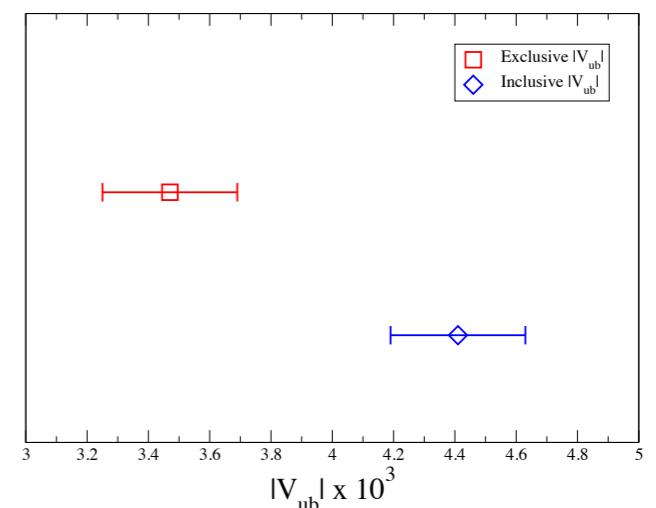


$B_s \rightarrow K \mu \nu$

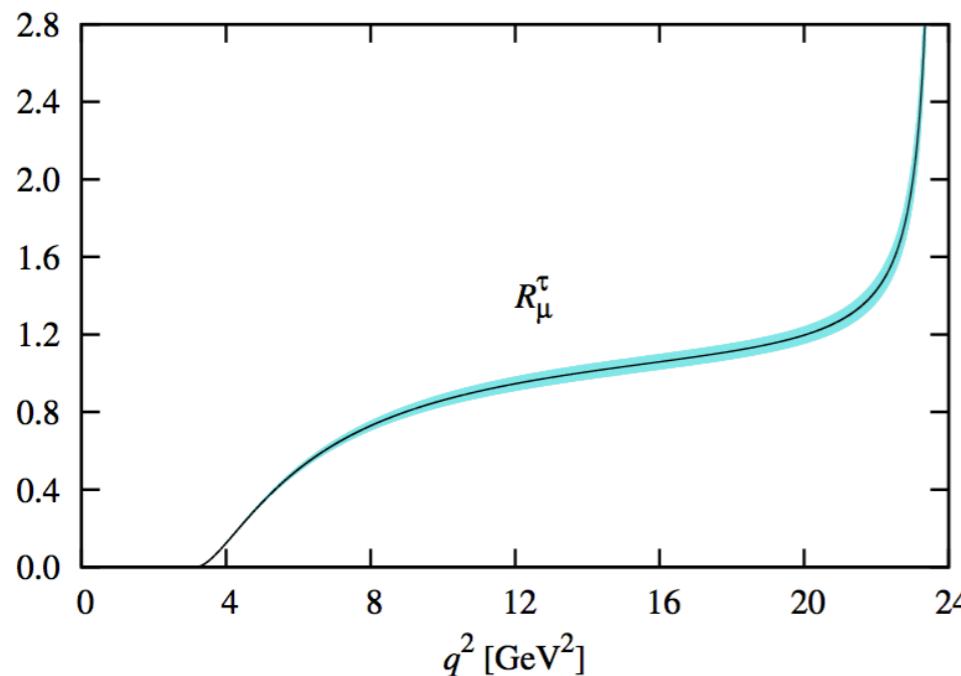


$B_s \rightarrow K \tau \nu$

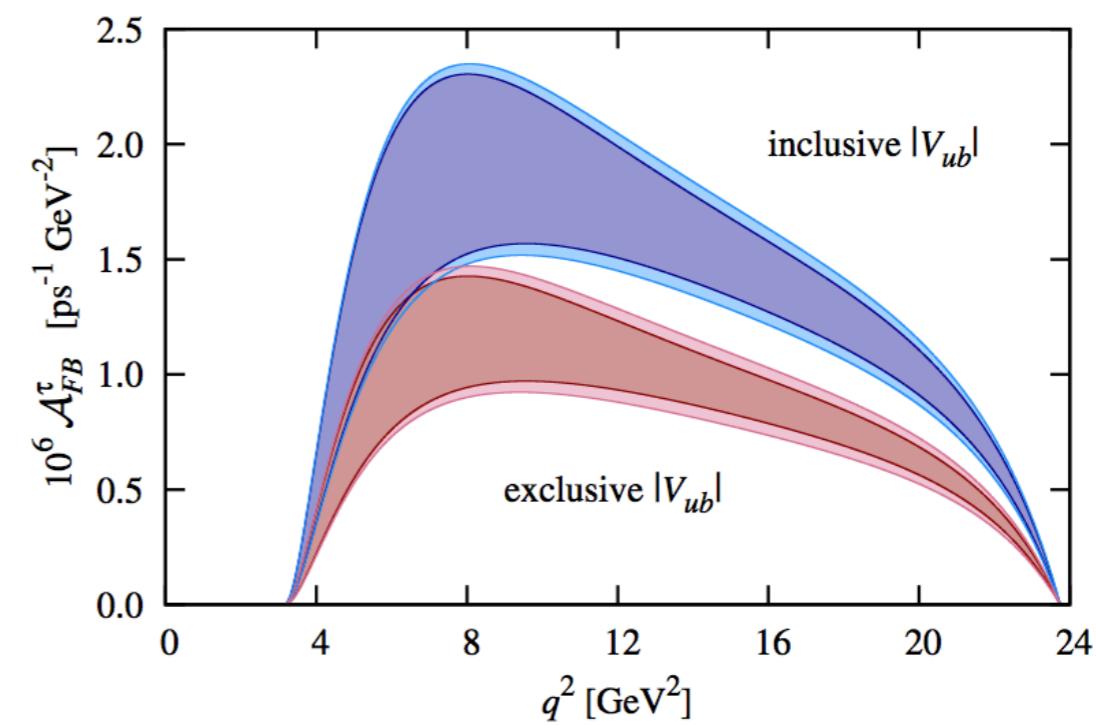
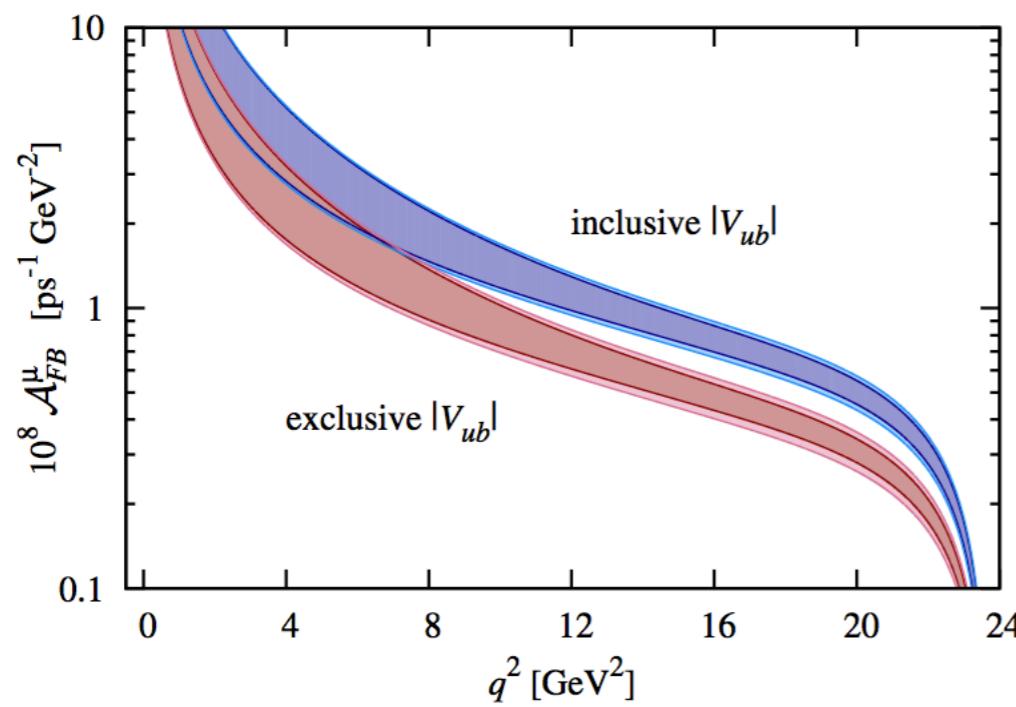
- Assuming inclusive or exclusive  $|V_{ub}|$
- Potential  $\sim 3 \sigma$  discrepancy at large  $q^2$
- Waiting for LHCb and BelleII



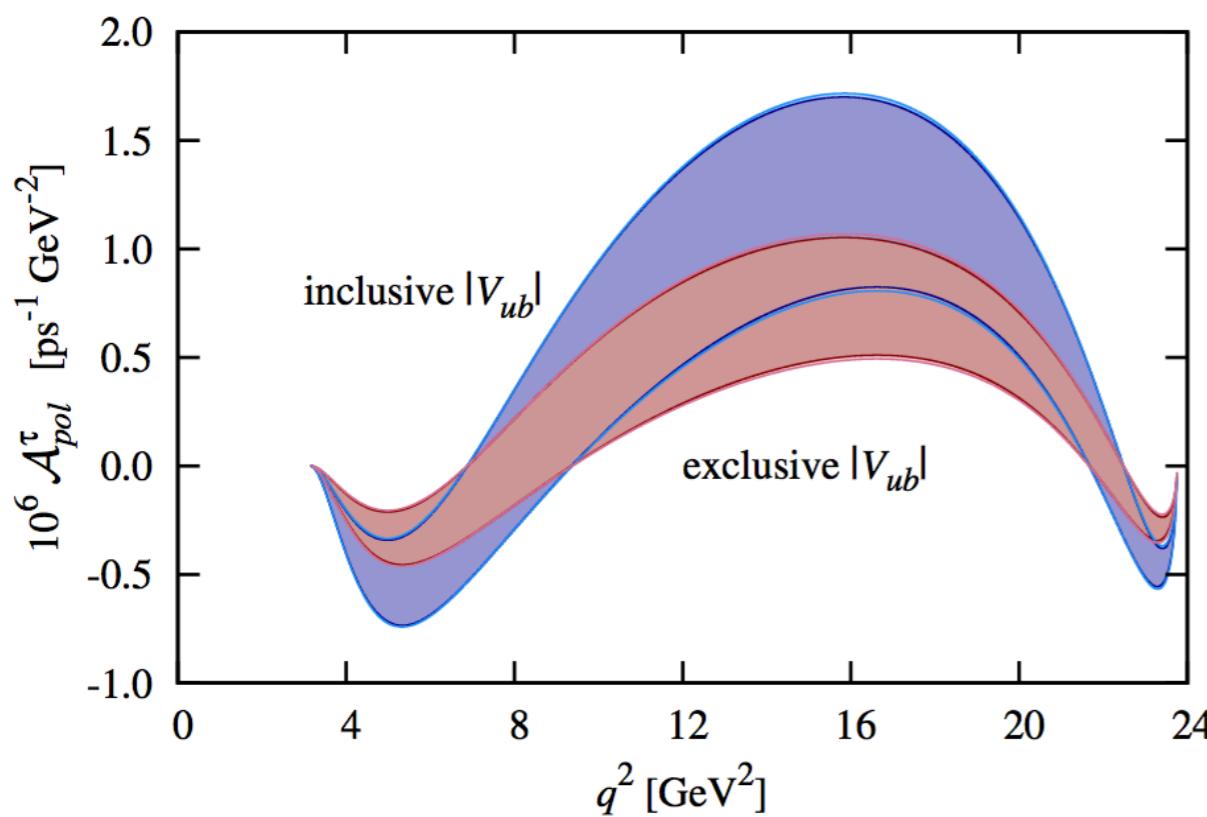
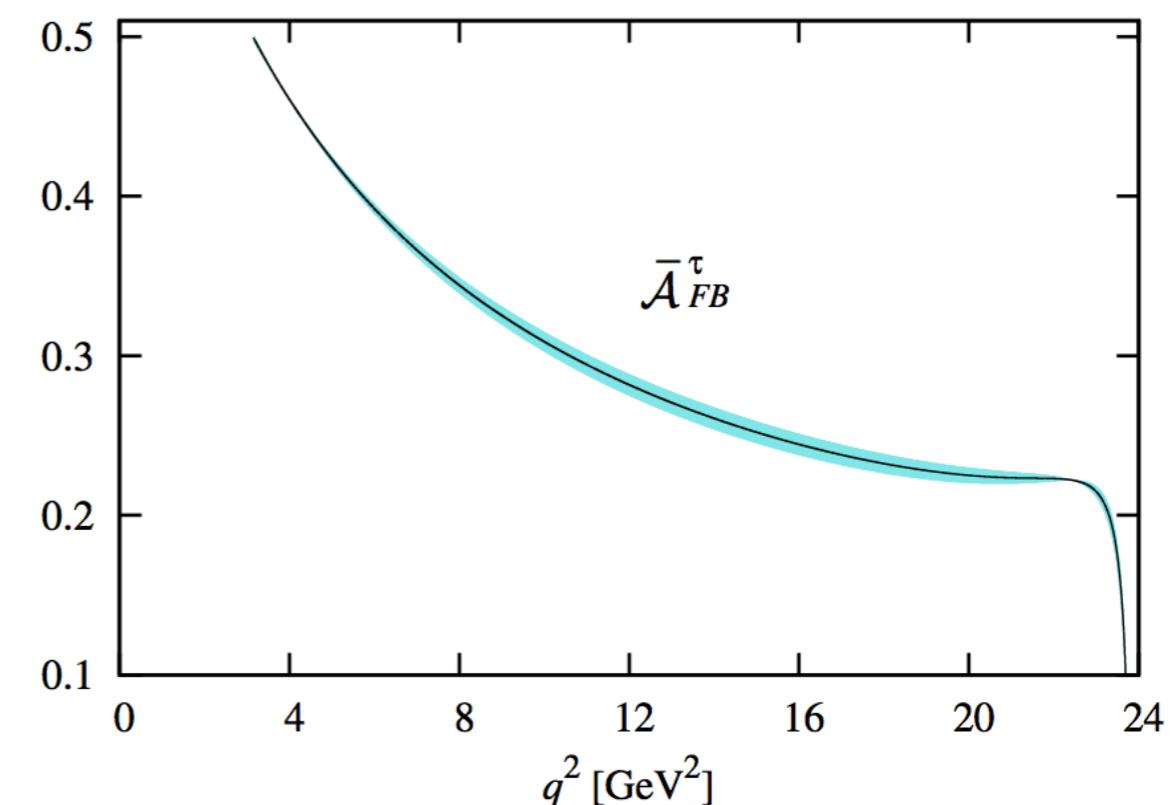
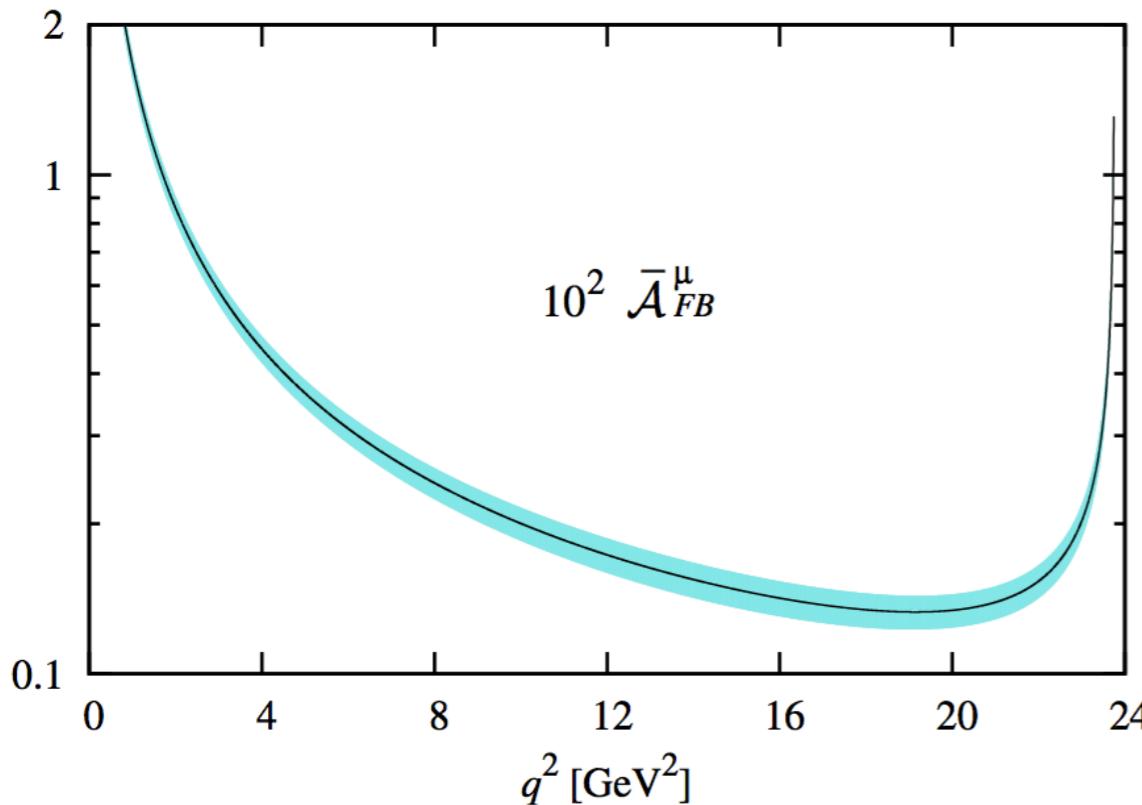
# Other phenomenological quantities



- $R_{\mu}\tau$  = Differential branching fraction ratio
- $A_{FB}$  = Forward and backward asymmetry

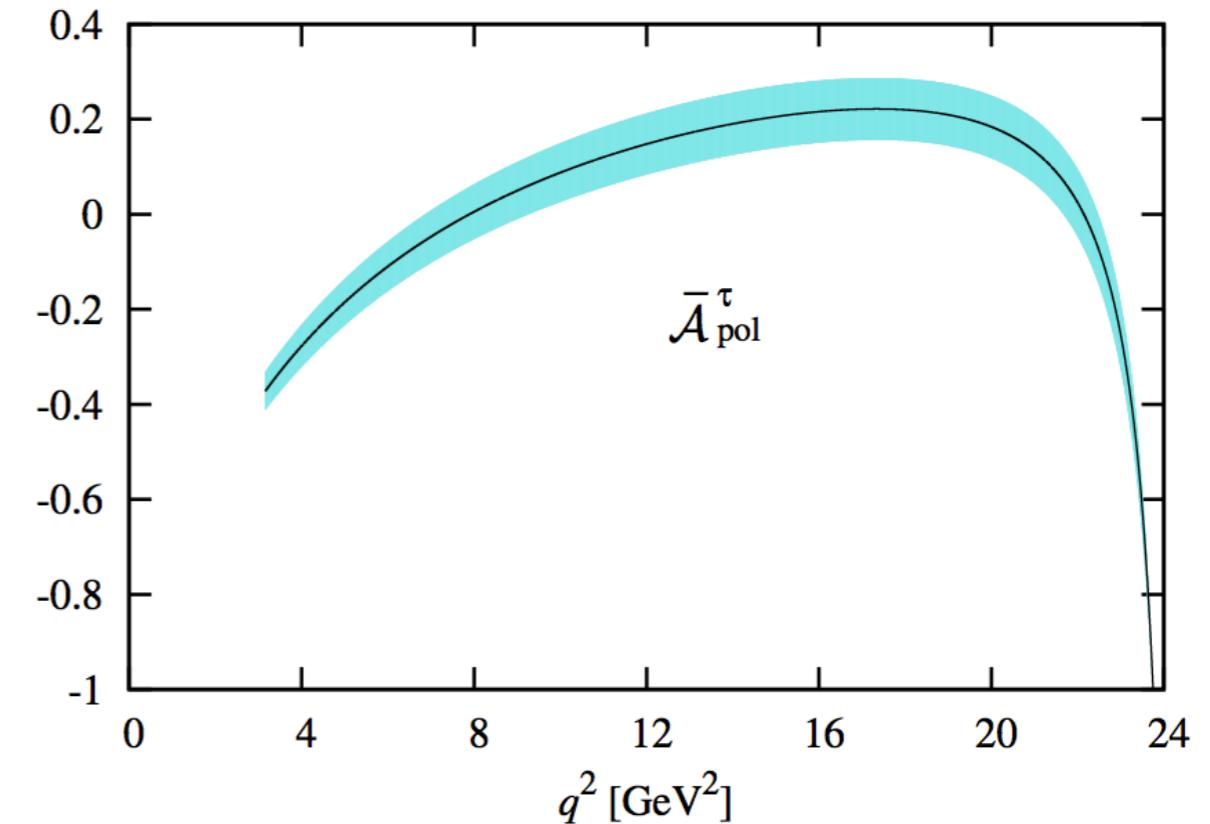
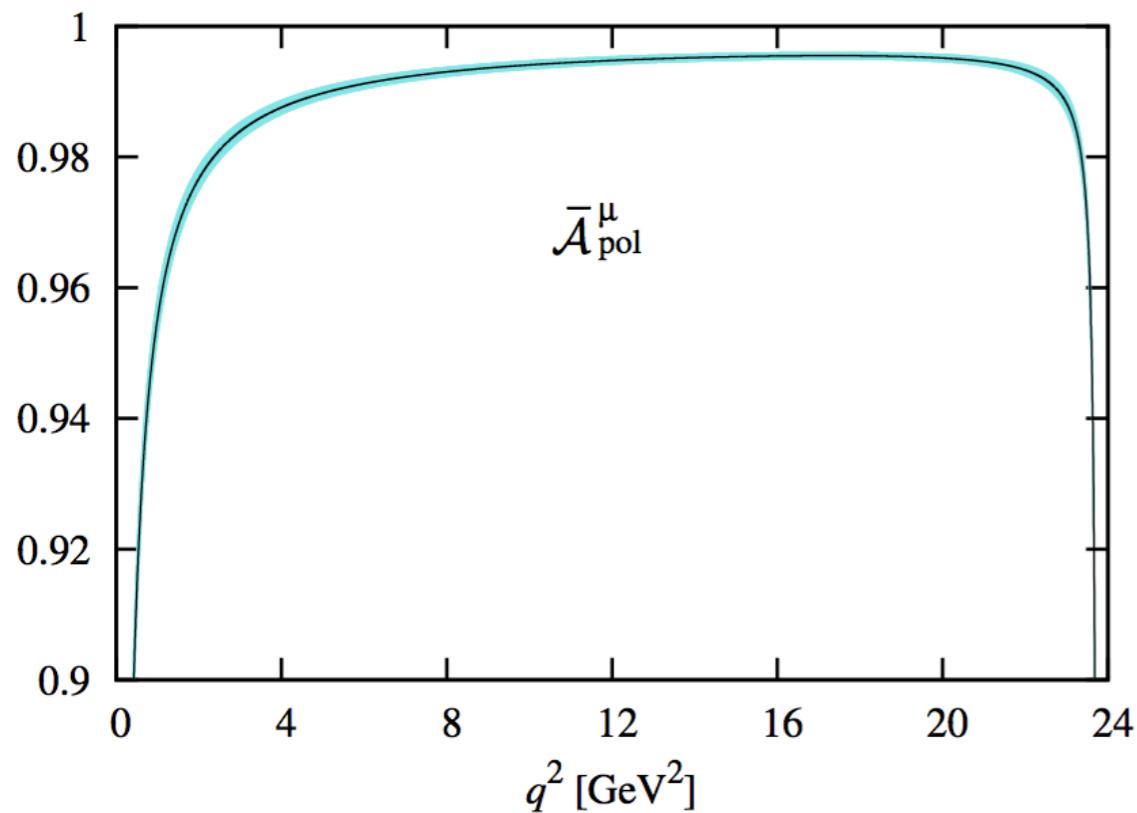


# Other phenomenological quantities



- $\bar{A}_{FB}$  = Normalized F-B asymmetry
- $A_{pol}\tau$  =  $\tau$  polarization distribution

## ◆ Other phenomenological quantities



- $\bar{A}_{\text{pol}}^\mu$  = Normalized  $\mu$  polarization distribution
- $\bar{A}_{\text{pol}}^\tau$  = Normalized  $\tau$  polarization distribution

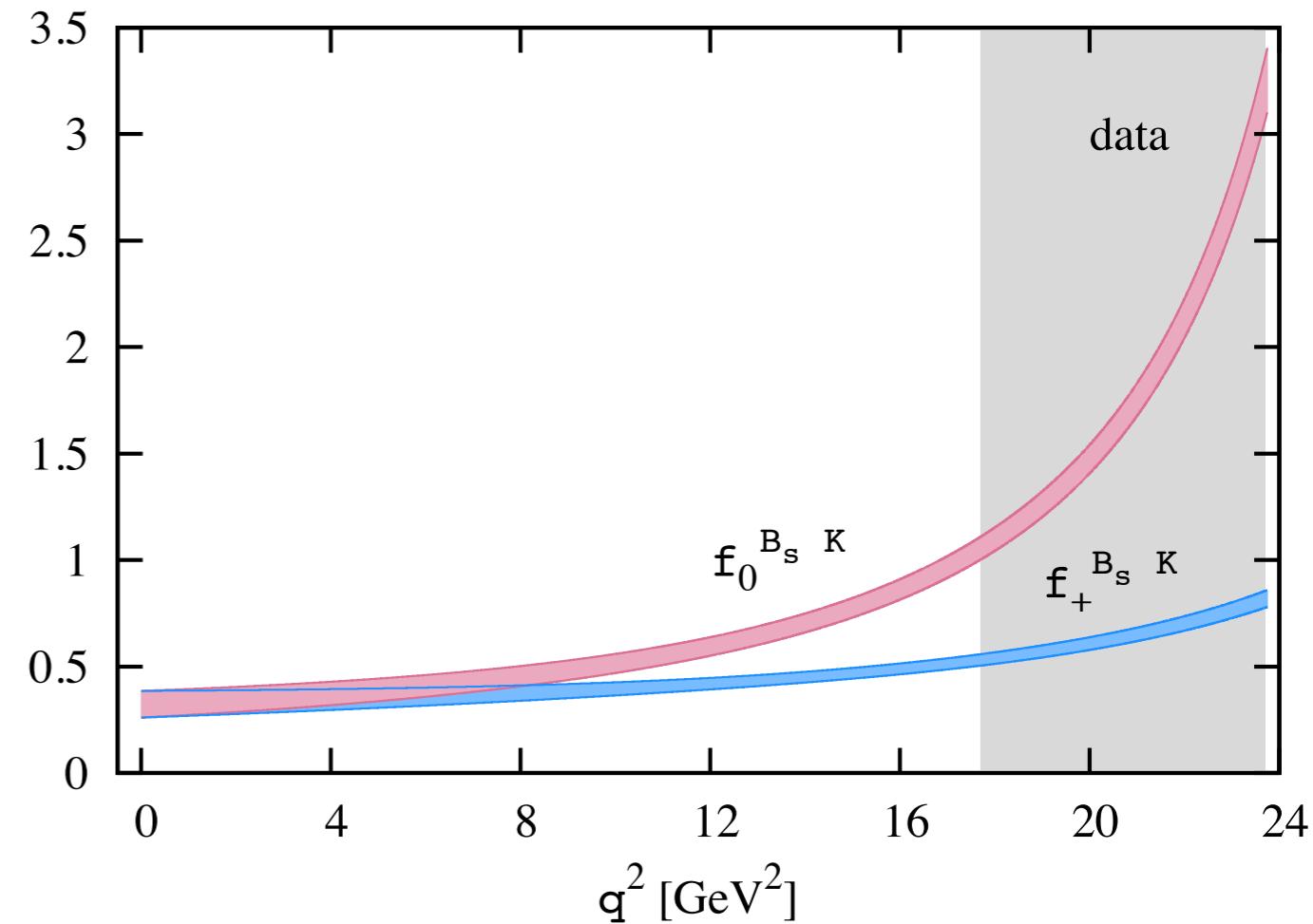
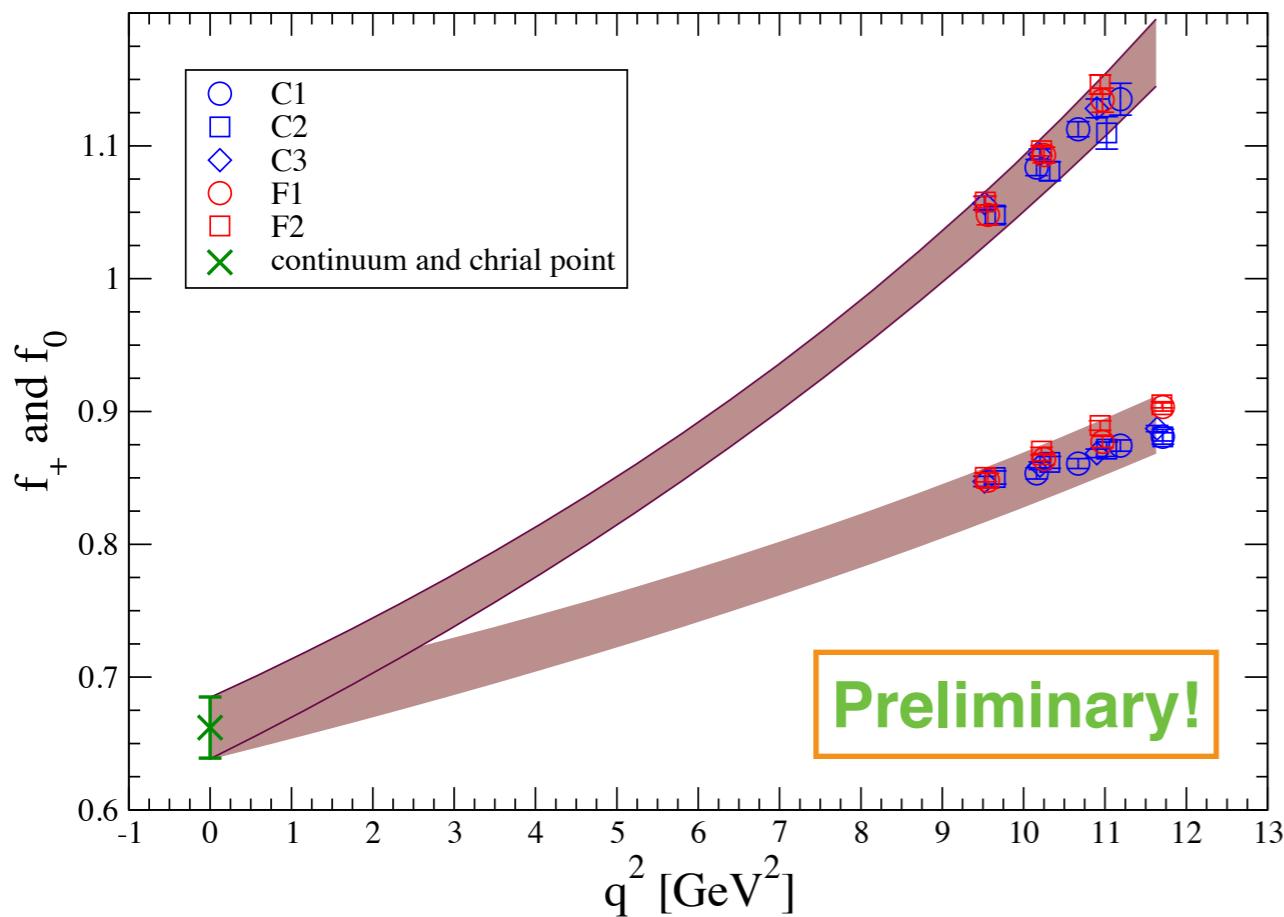
## Summary and future plan

---

- Our semileptonic programs have been very successful.
- We presented preliminary results on  $B \rightarrow D l \nu$  and  $B_s \rightarrow D_s l \nu$ , and complete analysis results on  $B_s \rightarrow K l \nu$
- $B \rightarrow D l \nu$  and  $B_s \rightarrow D_s l \nu$ 
  - HPChPT z-expansion
  - Correlations between  $B \rightarrow D$  and  $B_s \rightarrow D_s$  form factors
    - Ratio of the form factors  $\rightarrow \text{Br}(B_s \rightarrow \mu^+ \mu^-)$
- $B_s \rightarrow K l \nu$ 
  - Finished project: C. Bouchard et al., arXiv:1406.2279
  - Waiting experiment results from LHCb and BelleII
  - Precise non-perturbative renormalization for  $B \rightarrow \pi l \nu$  with  $B_s \rightarrow \eta_s l \nu$  with HISQ b quark

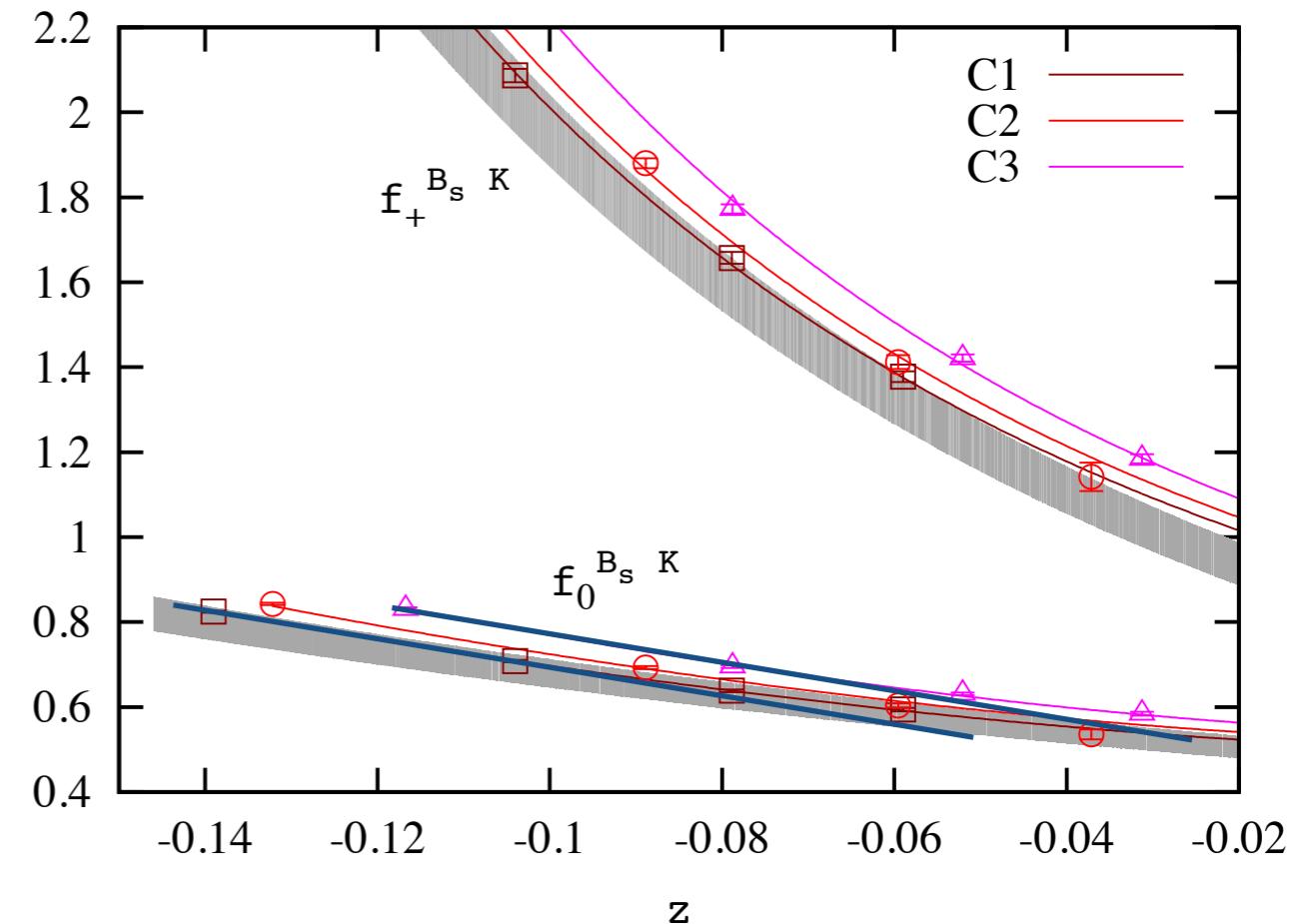
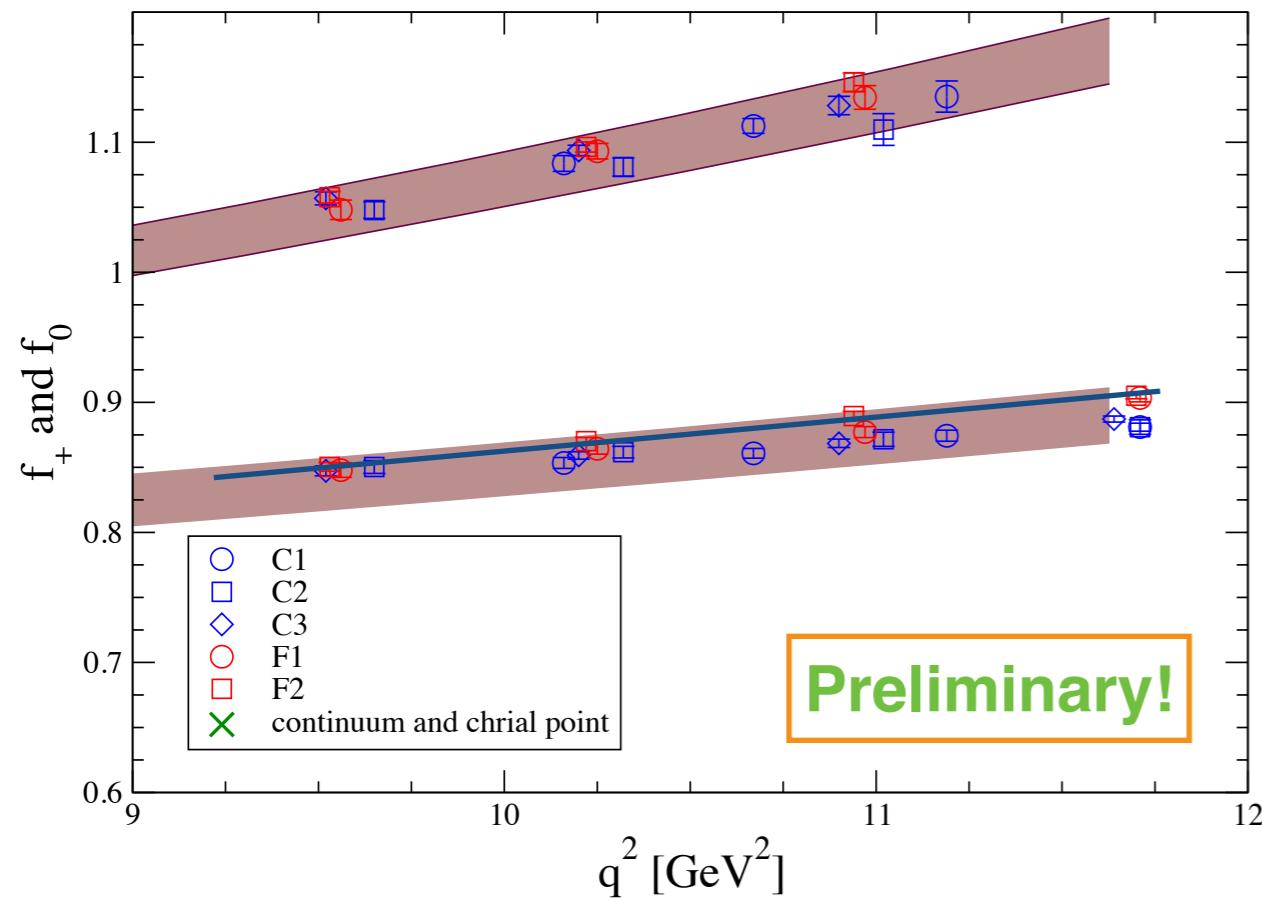
# Backup

# Pole for $f_0$



- $B_c^*(1^-)$ : 6.33 GeV → pole location: 40  $\text{GeV}^2$
- $B^*(1^-)$ : 5.325 GeV → pole location: 28.4  $\text{GeV}^2$

# Pole for $f_0$



- $B_c^*(1^-)$ : 6.33 GeV → pole location: 40  $\text{GeV}^2$
- $B^*(1^-)$ : 5.325 GeV → pole location: 28.4  $\text{GeV}^2$